

Level 1 / Level 2 GCSE (9 – 1)

MATHEMATICS

Paper 1 (Non- calculator)

Higher Tier

Time : 1 hour 30 minutes

Paper : 1 MA1 / 1H

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**



Information

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.



Answer ALL questions.

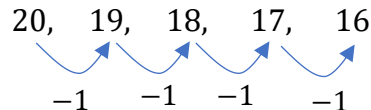
Write your answers in the spaces provided.

You must write down all the stages in your working.

1. The first five terms of an arithmetic sequence are

20 19 18 17 16

Write down an expression, in terms of n , for the n^{th} of this sequence.



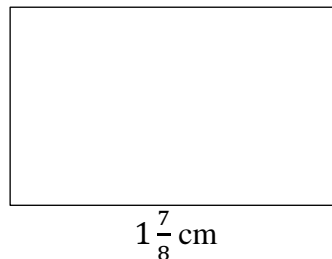
$$n^{\text{th}} \text{ term: } -n + x \quad (1)$$

$$1^{\text{st}} \text{ term: } -(1) + x = 20 \Rightarrow x = 21$$

$$n^{\text{th}} \text{ term: } -n + 21 \quad (1)$$

.....
 (Total for Question 1 is 2 marks)

- 2.



The diagram shows a rectangle.

The area of the rectangle is $6\frac{2}{3} \text{ cm}^2$.

The length of the rectangle is $1\frac{7}{8} \text{ cm}$.

Work out the width of the rectangle.

Give your answer in its simplest form.

$$6\frac{2}{3} = \frac{20}{3}$$

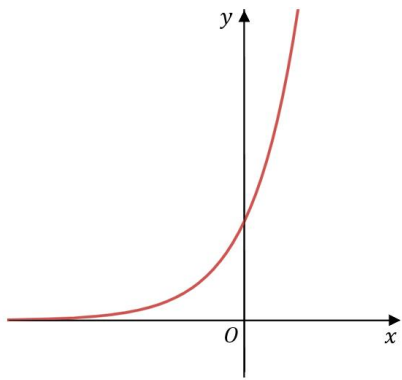
$$1\frac{7}{8} = \frac{15}{8} \quad (1)$$

$$w = \frac{20}{3} \div \frac{15}{8} = \frac{20}{3} \times \frac{8}{15} = \frac{160}{45} = \frac{32}{9} \quad (1)$$

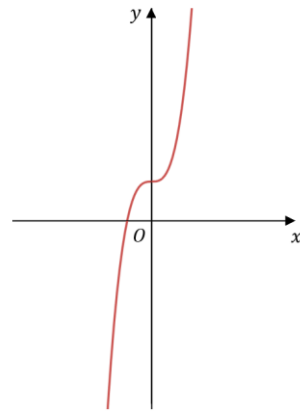
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 (Total for Question 2 is 3 marks)



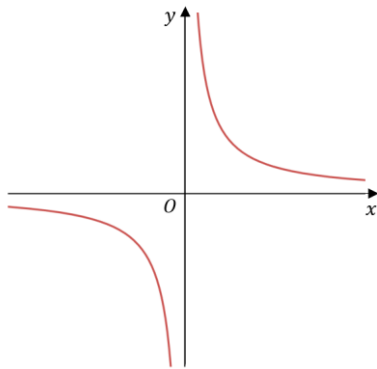
3.



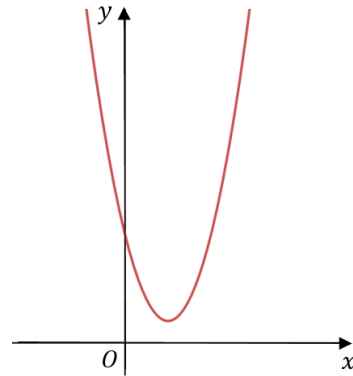
A



B



C



D

Each question in the table represents one of the graphs **A** to **D**.
Write the letter of each graph in the correct place in the table.

Equation	Graph
$y = x^2 - 4x + 5$	<i>D</i>
$y = 3 \times 2^x$	<i>A</i>
$y = x^3 + 2$	<i>B</i>
$y = \frac{2}{x}$	<i>C</i>

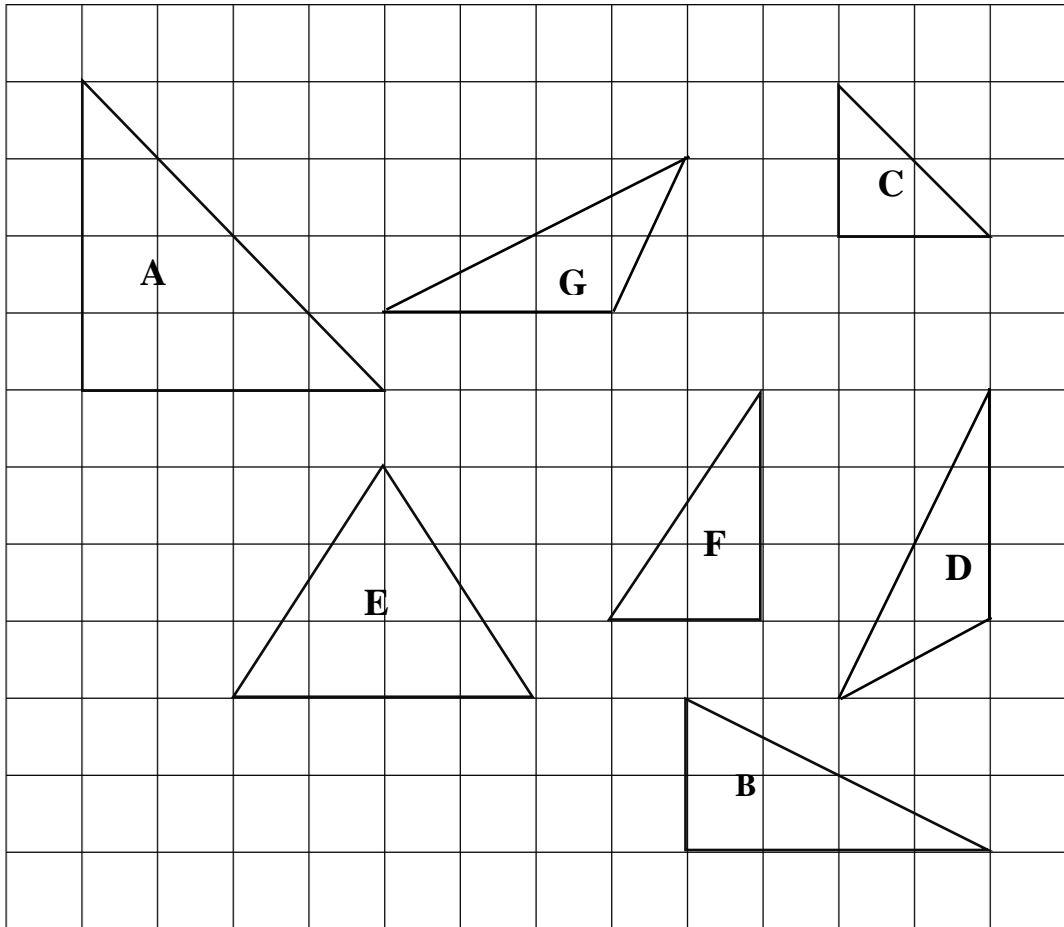
for two or three correct
all correct

(1)
(1)

.....
(Total for Question 3 is 2 marks)



4. Here are some triangles drawn on a square grid.



Two of the triangles are congruent.

a. Write down the letters of these two triangles.

correct answer only **D and G** (1)

..... and

(1)

One of the triangles is an enlargement of another of the triangles.

b. Write down the letters of these two triangles.

correct answer only **A and C** (1)

..... and

(1)

(Total for Question 4 is 2 marks)



5. Harry buys four tickets for a cinema.
Each ticket costs £12.50.
Harry also has to pay a booking fee.
The booking fee is £2.50 per ticket.
Work out the total amount Harry has to pay.

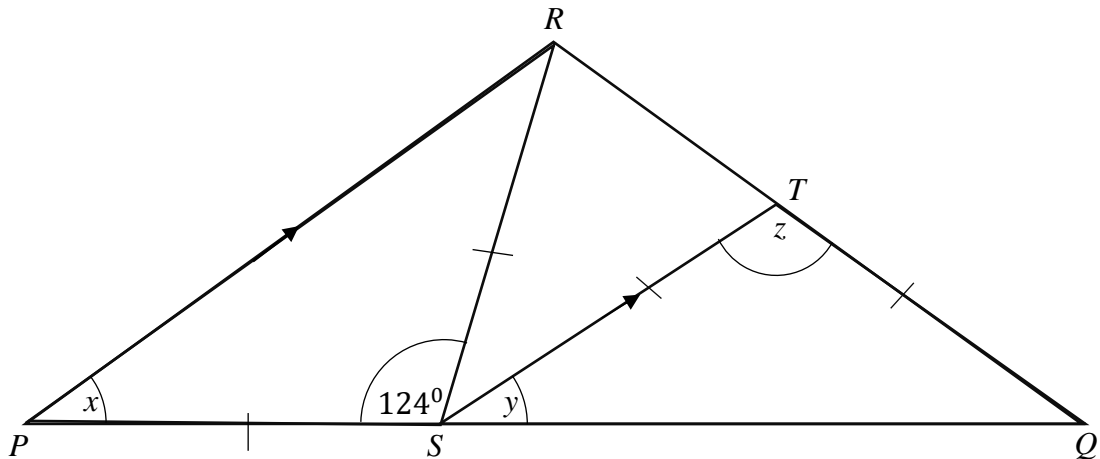
Total cost for one ticket:
 $£12.50 + £2.50 = £15$ (1)

Total cost for four tickets:
 $4 \times £12.50 = £60$ (1)

.....
(Total for Question 5 is 2 marks)



6.



PSQ is a straight line.

$PS = SR$

$ST = TQ$

Angle $PSR = 124^\circ$

- a. Work out the size of angle x , giving reasons for your answer.

Base angles in an isosceles triangle are equal, so angles $RPS = x$ and SRP are equal. (1)

Interior angles in a triangle total 180° , so $x = \frac{180-124}{2} = 28^\circ$ (1)

Must give justifications to obtain the marks.

.....^o
(2)

- b. Work out the size of angle y , giving reasons for your answer.

Angles x and y are corresponding angles. Corresponding angles are equal. (1)

$y = 28^\circ$ (1)

.....^o
(2)

- c. Work out the size of angle z , giving reasons for your answer.

Base angles in an isosceles triangle are equal, so angles y and SQT are equal. (1)

Interior angles in a triangle total 180° so $z = 180 - (28 \times 2) = 124^\circ$ (1)

.....^o
(1)

(Total for Question 6 is 5 marks)



7. A group of 20 Year 10 girls took a science test.
 The test was marked out of 50.
 The table gives information about their results.

	Median	Interquartile
Science	28	13

The stem and leaf diagram shows information about the Science test of a group of 23 Year 10 boys.

1		2 3 3 7 9
2		0 1 1 4 4 8
3		2 2 3 6 6 7 9
4		3 8 8
5		0 0

Use this information to compare the girls and boys science test results.

for identification of the interquartile of the boys

$$\text{e.g. } UQ - LQ = \left(\frac{23+1}{4} \times 3\right)^{\text{th}} - \left(\frac{23+1}{4}\right)^{\text{th}} = 39 - 20 = 19$$

or the median of the boys

$$\text{e.g. } \left(\frac{23+1}{2}\right)^{\text{th}} = 12^{\text{th}} = 32 \quad (1)$$

for a correct comparison of median or a correct comparison of interquartiles.

for a correct comparison of median and a correct comparison of interquartiles

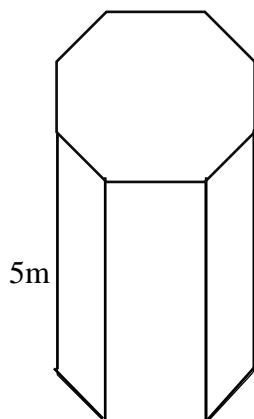
e.g. Boys have higher marks in Science. (1)

The spread of the Science test results is greater for boys i.e. Results are more consistent for Girls. (1)

(Total for Question 7 is 3 marks)



8. The diagram shows a prism.
 The prism is placed on a table so that the base area is in contact with the table.



$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The prism has height 5 m
 The prism exerts a force of 480 N (Newtons) on the table.
 The pressure on the table due to the prism is 80 N/m²

Work out the volume of the prism.

$$\begin{aligned}
 \text{area} &= \frac{\text{force}}{\text{pressure}} \\
 &= \frac{480\text{N}}{80\text{N/m}^2} \\
 &= 6\text{m}^2 \text{ (1)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Volume} &= \text{base} \times \text{height} \\
 &= 6 \times 5 \text{ (1)} \\
 &= 30\text{m}^3 = 30,000,000 \text{ cm}^3 \text{ (1)}
 \end{aligned}$$

.....cm³

(Total for Question 8 is 3 marks)

9. In the table below match the numbers written in standard form with their ordinary numbers.

7.28×10^5	↔	7280000
72.8×10^{-4}	↔	728000
728×10^4	↔	0.0000728
0.728×10^{-4}	↔	0.00728

for 2 or 3 correct matching (1)

for all correct matching (1)

(Total for Question 9 is 2 marks)



10. There are some red, blue and green balls in a bag.

The ratio of the number of red balls to the number of blue balls in a bag is 3 : 4 and the ratio of the number of blue balls to the number of green balls is 6 : 7.

Find the ratio of the number of red balls : blue balls : green balls.

$$\text{Red : Blue} = 3 : 4 = 9 : 12$$

$$\begin{array}{c} \curvearrowright \\ \times 3 \end{array}$$

$$\text{Blue : Green} = 6 : 7 = 12 : 14$$

$$\begin{array}{c} \curvearrowright \\ \times 2 \end{array}$$

$$\text{Red : Blue : Green} = 9 : 12 : 14$$

chooses a multiplier to equate the two fractions in terms of blue

or list equivalent fractions to $\frac{3}{4}$ up to at least $\frac{9}{12}$ e.g. $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \dots$

or list equivalent fractions to $\frac{6}{7}$ up to at least $\frac{12}{14}$ e.g. $\frac{6}{7}, \frac{12}{14}, \dots$

or for 9 : 12 or 12 : 14

(1)

puts into related terms ready for ratios e.g. $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ and $\frac{6}{7} \times \frac{2}{2} = \frac{12}{14}$

or for 9 : 12 and 12 : 14

or lists equivalent ratios up to a common element for blue

(1)

for red : blue : green = 9 : 12 : 14

(1)

.....

(Total for Question 10 is 3 marks)



11. a. Find the value of $\sqrt[3]{64 \times 10^6}$

$$\sqrt[3]{64 \times 10^6} = \sqrt[3]{64} \times \sqrt[3]{10^6}$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{10^6} = (10^6)^{\frac{1}{3}} = 10^2 = 100$$

$$\sqrt[3]{64 \times 10^6} = 4 \times 100 = 400$$

for working out $\sqrt[3]{64}$ as 4 or $\sqrt[3]{10^6}$ as 10^2 or 100 (1)

for 400 or 4×10^2 or 4×100 (1)

(2)

.....

c. Find the value of $27^{-\frac{1}{3}}$

$$27^{\frac{1}{3}} = \sqrt[3]{27} = 3 \quad (1)$$

$$27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{3} \quad (1)$$

(2)

.....

c. Write $\frac{2^n}{8^{n-2}}$ as a power of 2.

$$8^{n-2} = 2^{3(n-2)} \quad \text{or} \quad 2^{3n-6} \quad \text{or} \quad (2^3)^{n-2} \quad (1)$$

$$\frac{2^n}{8^{n-2}} = 2^n \div 2^{3(n-2)} = 2^{n-(3n-6)} = 2^{-2n+6} \quad (1)$$

(2)

.....

(Total for Question 11 is 6 marks)



12. The table gives information about the heights, in centimeters, of some plants.

Heights (h cm)	Frequency
$10 < h \leq 20$	16
$20 < h \leq 30$	24
$30 < h \leq 40$	44
$40 < h \leq 50$	52
$50 < h \leq 60$	20
$60 < h \leq 70$	16
$70 < h \leq 80$	8

a. Complete the frequency table.

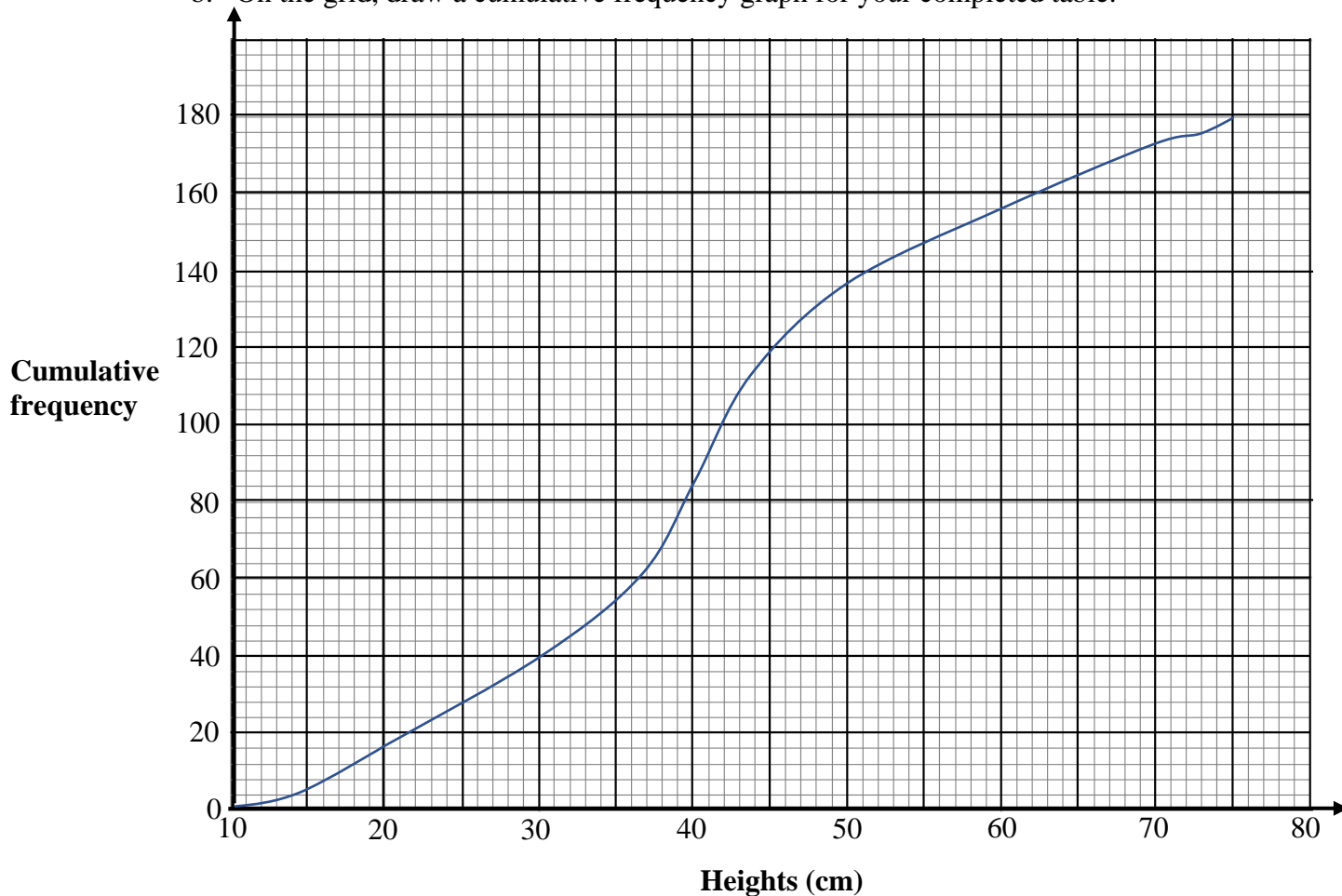
Heights (h cm)	Cumulative frequency
$10 < h \leq 20$	16
$10 < h \leq 30$	40
$10 < h \leq 40$	84
$10 < h \leq 50$	136
$10 < h \leq 60$	156
$10 < h \leq 70$	172
$10 < h \leq 80$	180

correct answer only

(1)

(1)

b. On the grid, draw a cumulative frequency graph for your completed table.



(2)



for 6 or 7 of their points plotted correctly from a table (1)

for a fully correct graph (1)

Pedro says

‘40% of this group of plants have a height of greater than 40 cm’

c. Is Pedro correct?

You must show how you get your answer.

84 plants have a height less than or equal to 40cm.

This means that $180 - 84 = 96$ plants have a height greater than 40cm. (1)

This corresponds to $\frac{96}{180} \times 100 = 53.\dot{3}\%$ (1)

Therefore, Pedro is incorrect, as 53.3% of this group of plants have a height of greater than 40 cm (1)

.....

(3)

(Total for Question 12 is 6 marks)

13. Claire makes a drink by mixing Liquid A with water.
She mixes 50 cm^3 of Liquid a with 450 cm^3 of water.

The density of Liquid A is 2 g/cm^3 .

The density of water is 1.00 g/cm^3 .

Work out the density of Claire’s drink.

$$\text{Mass of Liquid A} = 2 \text{ g/cm}^3 \times 50 \text{ cm}^3 = 100\text{g}$$

$$\text{Mass of water} = 1\text{g/cm}^3 \times 450\text{cm}^3 = 450\text{g}$$

$$\text{Density of Claire's drink} = \frac{100+450}{50+450} = 1.1\text{g/cm}^3$$

for density of Liquid A = 2×50 or density of water = 1×450 (1)

for density of Claire's drink = $\frac{100+450}{50+450} = \dots$ (1)

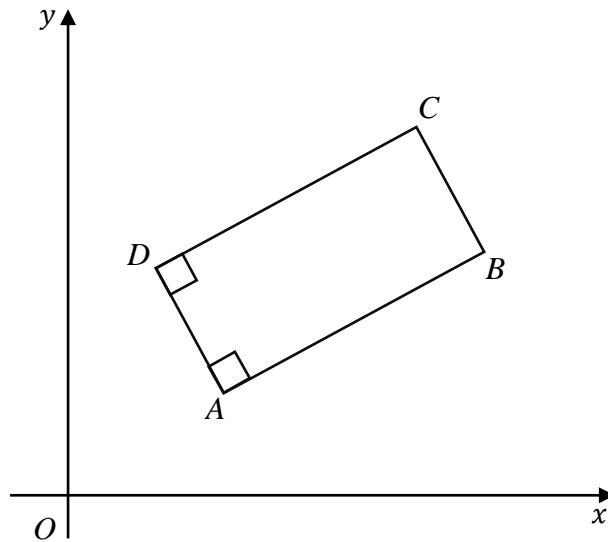
correct answer only 1.1 (1)

..... g/cm^3

(Total for Question 13 is 3 marks)



14.



$ABCD$ is a rectangle.

The equation of the straight line DA is $2x + y = 8$

The coordinates of C are $(5,10)$

Find the equation of the straight line CD .

$$DA: 2x + y = 8 \Rightarrow y = -2x + 8$$

The line CD is perpendicular to DA , so has gradient $-\frac{1}{(-2)} = \frac{1}{2}$. (1)

$DA: y = \frac{1}{2}x + c$. Substituting $(5,10)$ into this gives

$$10 = \frac{1}{2}(5) + c \Rightarrow c = \frac{15}{2} \text{ (1)}$$

$$y = \frac{1}{2}x + \frac{15}{2} \text{ (1)}$$

.....
(Total for Question 14 is 3 marks)



15. There are 5 blue pens, 3 green pens and 2 red pens in a box.
Gary takes at random a pen from the box and gives the pen to his friend.
Gary then takes at random another pen from the box.

Work out the probability that Gary will choose exactly one of the red pens.

For Gary to choose exactly one of the red pens, he will need to select Blue AND Red,
OR Green AND Red, OR Red AND Blue OR Red AND Green.

The probability of this is given by:

$$\left(\frac{5}{10} \times \frac{2}{9}\right) + \left(\frac{3}{10} \times \frac{2}{9}\right) + \left(\frac{2}{10} \times \frac{5}{9}\right) + \left(\frac{2}{10} \times \frac{3}{9}\right) = \frac{32}{90}$$

for appropriate multiplication e.g. BR , GR , RB or RG (1)

for complete process (1)

correct answer only $\frac{32}{90}$ or equivalent (1)

.....
(Total for Question 15 is 3 marks)



16. Martin wants to find an estimate for the number of white marbles in his bag.

On Saturday he picks 60 of the white marbles.
 He puts a red mark on each marble and replace them to his bag.

On Sunday he picks 90 of the white marbles.
 He finds that 15 of these white marbles have been marked.

a. Work out an estimate for the total number of white marbles in his bag.

$$\begin{aligned} \text{Population estimate} &= \frac{\text{No. in first sample} \times \text{No. in second sample}}{\text{No. in second sample previously marked}} \\ &= \frac{60 \times 90}{15} \\ &= 360 \end{aligned}$$

for $\frac{90}{15}$ or $\frac{15}{90}$ or $\frac{60}{15}$ or $\frac{15}{60}$ (1)

for $\frac{15}{90} = \frac{60}{n}$ or $\frac{15}{60} = \frac{90}{n}$ or $\frac{90 \times 60}{15}$ or equivalent (1)

correct answer only 360 (1)

.....
(3)

Martin assumes that none of the red marks had rubbed off between Saturday and Sunday.

b. If Martin's assumption is wrong, explain what effect this would have on your answer to part (a).

acceptable examples e.g.
 if marks fall off Martin will have overestimated the number of marbles
 there will be fewer marbles
 the total number of marbles will go down (1)

not acceptable examples e.g.
 My answer will be wrong
 It will increase the answer.

.....
(1)

(Total for Question 16 is 4 marks)



17. Make y the subject of the formula

$$x = \frac{y}{y-3}$$

$$x = \frac{y}{y-3} \Rightarrow x(y-3) = y \Rightarrow xy - 3x = y \Rightarrow xy - y = 3x \Rightarrow y(x-1) = 3x$$

(1)

(1)

(1)

$$y = \frac{3x}{x-1} \quad (1)$$

.....
(Total for Question 17 is 4 marks)

18. p is proportional to \sqrt{q} where $q > 0$.

q is decreased by 36%.

Work out the percentage decrease in p .

$$p = k\sqrt{q} \quad (1)$$

$$p = k\sqrt{0.64q} = 0.8k\sqrt{q} \quad (1)$$

So there is a 20% decrease in p (1)

.....
(Total for Question 18 is 3 marks)



19. f and g are functions such that

$$f(x) = \frac{x+1}{3} \quad g(x) = \sqrt{x-2}$$

a. Find $f(11)$

$$f(11) = \frac{11+1}{3} = 4 \quad (1)$$

.....
(1)

b. Find $fg(27)$

$$g(27) = \sqrt{27-2} = 5$$

$$fg(27) = f(5) = \frac{5+1}{3} = 2$$

for $g(27) = \sqrt{27-2} = 5$ and clear intention to find $f(5)$

or for $\frac{\sqrt{27-2}+1}{3}$ (1)

correct answer only 2 (1)

.....
(2)

c. Find $f^{-1}(3)$

$$\frac{x+1}{3} = 3 \Rightarrow x = 8$$

for f^{-1} as $3x - 1$ or stating to solve $\frac{x+1}{3} = 3$ (1)

correct answer only 8 (1)

.....
(2)

(Total for Question 19 is 5 marks)

20. Show that $\frac{\sqrt{216}+3\sqrt{6}}{3\sqrt{3}-3\sqrt{2}}$ can be written in the form $(a\sqrt{2} + b\sqrt{3})$ where a and b are integers.

$$\sqrt{216} = \sqrt{36 \times 6} = 6\sqrt{6} \quad (1)$$

$$\frac{\sqrt{216}+3\sqrt{6}}{3\sqrt{3}-3\sqrt{2}} = \frac{6\sqrt{6}+3\sqrt{6}}{3\sqrt{3}-3\sqrt{2}}$$

$$= \frac{9\sqrt{6}}{3\sqrt{3}-3\sqrt{2}} \times \frac{3\sqrt{3}+3\sqrt{2}}{3\sqrt{3}+3\sqrt{2}} \quad (1)$$

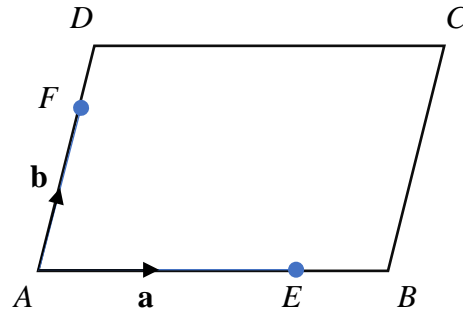
$$= \frac{3\sqrt{3} \times \sqrt{216} + 3\sqrt{2} \times \sqrt{216} + 3\sqrt{6} \times 3\sqrt{3} + 3\sqrt{6} \times 3\sqrt{2}}{27-18} = 3\sqrt{18} + 3\sqrt{12} \quad (1)$$

$$= 9\sqrt{2} + 6\sqrt{3} \quad (1)$$

(Total for Question 20 is 4 marks)



21. $ABCD$ is a parallelogram.



E is the point on AB such that $EB = \frac{1}{4}AB$

F is the point on AD such that $FD = \frac{1}{4}AD$

$$\overrightarrow{AB} = \mathbf{a}$$

$$\overrightarrow{AD} = \mathbf{b}$$

a. Find in terms of \mathbf{a} and/or \mathbf{b}

i. \overrightarrow{DB}

$$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = -\mathbf{b} + \mathbf{a} \quad (1)$$

(1)

ii. \overrightarrow{FE}

$$\overrightarrow{FE} = \frac{3}{4}(\overrightarrow{FA} + \overrightarrow{AB}) = \frac{3}{4}(-\mathbf{b} + \mathbf{a}) \quad (1)$$

(1)

b. Show that FE is parallel to DB .

for complete proof with statements and a conclusion

$$\text{e.g. } FE = kDB \Rightarrow \frac{3}{4}(-\mathbf{b} + \mathbf{a}) = k(-\mathbf{b} + \mathbf{a}) \Rightarrow k = \frac{3}{4} \quad (1)$$

$$FE = \frac{3}{4}DB \quad \text{so } FE \text{ is parallel to } DB \quad (1)$$

(2)

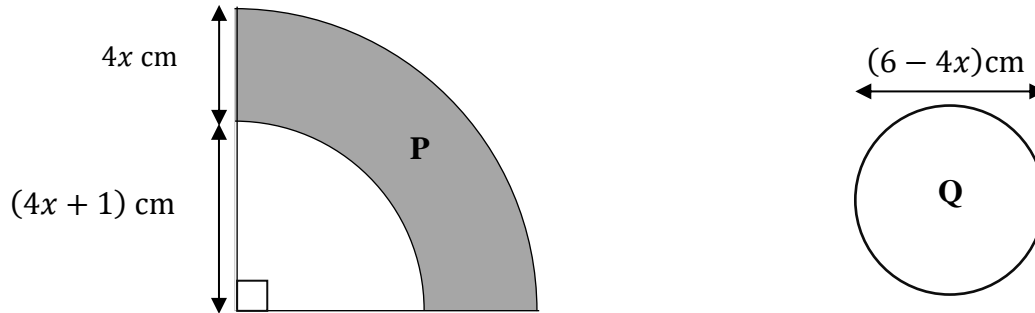
.....
 (Total for Question 21 is 4 marks)



22. The diagram shows two shaded shapes **P** and **Q**.

Shape **P** is formed by removing a quarter of a circle with radius $(4x + 1)$ cm from a quarter of a circle with radius $(8x + 1)$ cm.

Shape **Q** is a circle of diameter $(6 - 4x)$ cm.



The area of shape **P** is equal to the area of shape **Q**.

Find the value of x .

You must show all your working.

$$P = \frac{1}{4}\pi[(8x + 1)^2 - (4x + 1)^2] \quad (1)$$

$$Q = (3 - 2x)^2\pi$$

$$P = Q \Rightarrow \frac{1}{4}\pi(48x^2 + 8x) = \pi(9 - 12x + 4x^2) \Rightarrow 8x^2 + 14x - 9 = 0 \quad (1)$$

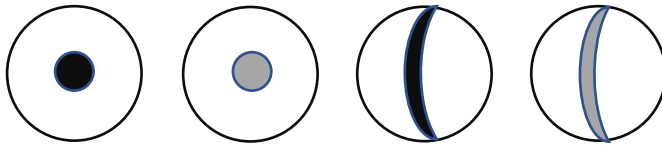
$$x = \frac{-14 \pm \sqrt{14^2 - 4 \times 8 \times -9}}{2 \times 8} \Rightarrow x = -2.25, 0.5 \quad (1)$$

A distance cannot be negative, so $x = \frac{1}{2}$ (1)

.....
(Total for Question 22 is 5 marks)



23. In a competition four types of balls have been oriented in a bag.
 Each ball has a black spot or grey spot or red stripe or a yellow stripe.



Number of balls with a black colour : number of balls with a grey colour = 4 : 5

Number of balls with a spot : number of balls with a stripe = 2 : 9

Express the total number of balls with a black colour as a fraction of the total number of balls with stripe.

$$\text{No of balls with black} = \frac{4}{9}$$

$$\text{No. of balls with stripe} = \frac{9}{11}$$

$$\text{Fraction is given by } \frac{4}{9} \div \frac{9}{11} = \frac{4}{9} \times \frac{11}{9} = \frac{44}{81}$$

for $\frac{4}{9}$ and $\frac{9}{11}$

or uses a total of 99 balls and shows a process to find the number of balls with a black colour or the number of balls with stripe (1)

for process shown to divide fractions

or uses a total of 99 balls and shows a process to find the number of balls with a red balls and the number of balls with stripe (1)

for $\frac{44}{81}$ or any other equivalent fraction (1)

.....
 (Total for Question 23 is 3 marks)

TOTAL FOR PAPER IS 80 MARKS

