Mark schemes

1. (a) Relating the gravitational force to mass × acceleration with the acceleration being centripetal in any form ✓1

$\frac{GMm}{r^2} = (ma) = \frac{mv^2}{r}$ *or* $mr\omega^2$ *or* $mv\omega$ *and r can be replaced by* $R + h$

show the working leading to answer = $\omega = \sqrt{\frac{GM}{(R+h)^3}}$ ✓2

2

(b) $\left(\omega = \sqrt{\frac{GM}{(R+h)^3}} = \sqrt{\frac{6.67\times10^{-11}\times5.97\times10^{24}}{(2.02\times10^{7}+6.37\times10^{6})^3}}\right)$

✓1 A full substitution in the equation can gain this mark even if ω *is incorrectly calculated*

= 1.45×10^{-4} (rad s^{-1}) ✓1

$T = \frac{2\pi}{\omega} = \frac{2\pi}{1.45\times10^{-4}} = 4.3(1)\times10^{4}$ (s) ✓2

A final correct answer for T *gains both marks*

2

(c) Launch from **Z** with some speed or energy argument ✓1

✓1 Condone ref. to equator rather than Z

At this position the satellite has the largest initial speed/kinetic energy from the Earth's rotation. ✓2

Consider answer Y only if <u>extremely</u> well explained in terms of different potentials and fuel use.

2

(d) gravitational potential energy

$= (-)\frac{GMm}{r}$ OR $(-)\frac{6.67\times10^{-11}\times5.97\times10^{24}\times1630}{2.66\times10^{7}}$ ✓1

gravitational potential energy = -2.44×10^{10} (J) ✓2

✓1 for the equation or substitution without the need for the negative sign

only allow ecf for r = 6.37×10^{6}

giving gpe = -1.02×10^{11} *J*

✓2 the minus is necessary for the mark

Correct answer gains both marks

2

(e) In a higher orbit the linear speed is smaller using

$v = \sqrt{\frac{GM}{r}}$ or $v \propto \frac{1}{\sqrt{r}}$ ✓1

Suitable justification of $v \propto \frac{1}{\sqrt{r}}$ ✓2

✓2 example

$$v = r\omega = r\sqrt{\frac{GM}{(R+h)^3}} = r\sqrt{\frac{GM}{(r)^3}} = \sqrt{\frac{GM}{r}}$$

Or derivation from basics

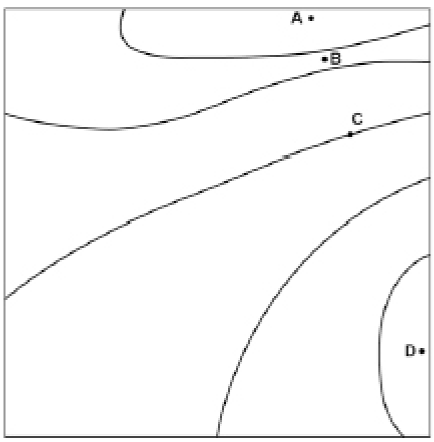
2

[10]

2. **A**

[1]

3. B



[1]

4. (a) $\frac{GmM}{r^2}(= ma) = mr\omega^2$

OR $\frac{GMm}{r^2} = \frac{mv^2}{r}$ ✓

The starting point must be from relating forces or accelerations – not a remembered equation produced at a later stage in the manipulation of the equation.

$\omega^2 = \left(\frac{2\pi}{T}\right)^2$

OR $v = \frac{s}{t} = \frac{2\pi r}{T}$ ✓

Middle mark may be given when seen as a substitution and can be a stand-alone mark.

$\frac{GM}{r^2} = r\frac{4\pi^2}{T^2}$ results in $T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$ ✓

Last mark is only given if accompanied with the working and must eventually be in the form with T^2 and r^3.

(b) $\left(\frac{T_U^2}{T_M^2} = \frac{r_U^3}{r_M^3}\right)$

$$r_U = r_M \left(\frac{T_U}{T_M}\right)^{2/3}$$

OR

Substitution of data in equation in any of its forms ✓

The first mark is for converting the equation into a proportion with or without substitution of data.

Eg $r_U = 1.29 \times 10^8 \left(\frac{4.14}{1.41}\right)^{2/3}$

(r_U=) radius = 2.6(5) x 10^8 (m) ✓

Last mark is for evaluating the correct answer

Answer only gains both marks

2

(c) Rearranging

$$T^2 = \frac{4\pi^2}{GM}\ r^3$$

To give

$$M = \frac{4\pi^2}{T^2 G}\ r^3 \quad \text{or } M = \frac{4\pi^2}{Gk}$$

Or substitution of data ✓(allow use of their calculated k from (b))

Conversion of T to seconds for Miranda

$T = 1.41 \times 24 \times 3600 = 1.22 \times 10^5$ s ✓ (or $T^2 = 2.48 \times 10^{10}$)

Substitution into equation and determination of M

$M = 8.5(6) \times 10^{25}$ kg ✓

Give full credit for use of Umbriel period and their answer to (b)

Alternative:

Conversion of T to seconds for Umbriel

$T = 4.14 \times 24 \times 3600 = 3.58 \times 10^5$ s ✓

Determine k from

$T^2 = kr^3$ for Miranda (or Umbriel) ✓

$k = 6.9 \times 10^{-15}$

Converting to seconds mark stands alone

use of k = $k = \frac{4\pi^2}{GM}$

to find $M = 8.56 \times 10^{25}$ kg✓

No ecf for final answer mark.

3

(d) $\frac{GM\cancel{m}}{(d/2)} = \frac{1}{2}\cancel{m}v^2$

OR

$v = \left(\frac{4GM}{d}\right)^{1/2}$

OR

stating the escape velocity depends on M/d ✓

Any correct calculation of an escape velocity or ratio M/d or $\left(M/d\right)^{1/2}$ ✓

Last mark only given with three correct relevant calculations

Correct conclusion = Titania ✓

Full credit may be given for answers that use r rather than d.

Table to help identify 2nd mark

Name	m/d	(m/d)^{½}	v
Ariel	1.09×10^{15}	3.31×10^{7}	540
Oberon	1.99×10^{15}	4.46×10^{7}	729
Titania	2.21×10^{15}	4.70×10^{7}	768

3

(e) (On Ariel surface $g = \frac{GM}{r^2} = \frac{6.67\times10^{-11}\times1.27\times10^{21}}{(1.16\times10^6/2)^2}$)

g = 0.25 (m s^{-2}) ✓

(spring supplies the same potential energy so

$(\cancel{m}gh)_{\text{Earth}} = (\cancel{m}gh)_{\text{Ariel}}$)

9.8 × 1.0 = 0.25 × h

OR

(comparing $\frac{v^2-u^2}{2a}$

$h = \frac{0-20}{-2\times0.25}$ ✓

h = 39 or 40 m , so it could not ✓ {allow ecf for arithmetic errors only}

The second mark can be given for the idea that the gravitational potential energy gained on the Earth is the same as that on Ariel with or without data.

Or that the initial kinetic energy is the same in both cases.

Last mark allow ecf for arithmetic errors only.

Condone incorrect use of signs.

3

[14]

5. B

4g

[1]

6. A

7.2 J

[1]

7. (a) the work done/energy <u>required</u> in bringing <u>1 kg/unit</u> mass from infinity to the point ✓

A test mass should not be taken to be a unit mass.

Ignore extra comments eg about charge.

1

(b) The potential difference between the lines is constant but the distances are not

Or

$\frac{\Delta V}{\Delta r}$ is changing ✓ (potentials $\frac{\Delta V}{\Delta r}$,inner 0.857, outer = 0.625)

Or

The equipotential surfaces are not straight / not parallel / are curved ✓

The mark is given for the idea that the separation should be uniform or that equipotential lines should not be curved. Owtte Discussions should not imply there is a correct curvature.

Errors can come from references to the moon to first equipotential distance. Or by saying the potential gaps are not uniform or by saying the distance from the centre of the Moon is not proportional to the potential.

1

(c) M = mass of Moon = $= \frac{-V \times r}{G} = \frac{(-1) \times -1.60 \times 10^6 \times 3.06 \times 10^6}{6.67 \times 10^{-11}}$ ✓$_1$

✓$_1$ The mark is given for use and rearranging the equation so errors may be seen in the data and any equipotential may be used.

Condone the misuse of a negative sign

$M = 7.3$ or 7.4×10^{22} (kg) ✓$_2$

✓$_2$ Note All equipotential lines produce the same mass.

The answer may be seen in the equations that follow.

(Use of $\frac{GMm}{r} = \frac{1}{2}mv^2$)

$v = \sqrt{\frac{2GM}{r}}$ ✓$_3$

✓$_3$ An attempt to use this re-arranged formula gains this 3rd mark.

v = 2.3 or 2.4 × 10^3 (m s^{-1}) ✓$_4$ (2370 m s^{-1})

Alternative

(Use of $V = \frac{-GM}{r}$)

$Vr = -MG$ = constant
$Vr = -1.60 \times 10^6 \times 3.06 \times 10^6$
$= -4.9 \times 10^{12}$ (J Kg^{-1} m) ✓$_{1Alt}$

✓$_{1Alt}$ Any attempt to calculate Vr OR to indicate that it is constant gains this mark.

(at the surface of the Moon)

$V_{surface} = \frac{-4.9 \times 10^{12}}{1.74 \times 10^6}$

$V_{surface} = -2.8(2) \times 10^6$ (J kg^{-1}) ✓$_{2Alt}$

(Use of mV = $\frac{1}{2}mv^2$)

$\sqrt{2V_{surface}} = \sqrt{2 \times 2.82 \times 10^6}$ ✓$_{3Alt}$

✓$_{3Alt}$ An attempt to use this re-arranged formula gains this 3rd mark.

v = 2.4×10^3 (m s^{-1}) ✓

✓$_{4Alt}$ A correct answer shows the previous work has been done correctly and gains all 4 marks.

4

[6]

8. **D**

[1]

9. **B**

[1]

10. **D**

[1]

11. (a) (centripetal) force = $m\,r\,(2\pi / T)^2$ Or $m\,r\,(\omega)^2$
(is given by the gravitational) force = $G\,m\,M / r^2$ ✓ (mark for both equations)
(equating both expressions and substituting for ω if required) $T^2 = (4\pi^2 / GM)\,r^3$ ✓ ($4\pi^2 / GM$ is constant, the constants may be on either side of equation but T and r must be numerators)

First mark is for two equations (gravitational and centripetal)

The second mark is for combining.

2

(b) (use of $T^2 \propto r^3$ so $(T_P / T_E)^2 = (r_P / r_E)^3$)
$(T_P /1.00)^2 = (5.91 \times 10^9 /1.50 \times 10^8)^3$ ✓ (mark is for substitution of given data into any equation that corresponds to the proportional equation given above)
$(T_P^2 = 61163)$
$T_P = 250$ (yr) ✓ (247 yr)

Answer only gains both marks

The calculation may be performed using data for the Sun in $T^2 = (4\pi^2 / GM)\, r^3$ easily spotted from $M_s = 1.99 \times 10^{30}$ kg giving a similar answer 247 – 252 yr.

2

(c) using $M (= g\, r^2 / G) = 0.617 \times (1.19 \times 10^6)^2 / 6.67 \times 10^{-11}$ ✓
$M = 1.31 \times 10^{22}$ kg ✓
answer to 3 sig fig ✓ (this mark stands alone)

The last mark may be given from an incorrect calculation but not lone wrong answer.

3

(d) Initial KE = ½ (m) $1400^2 = 9.8 \times 10^5$ (m) J✓
Energy needed to escape = 7.4×10^5 (m) J ✓
So sufficient energy to escape. ✓

OR For object on surface escape speed given by $7.4 \times 10^5 = ½\, v^2$
✓
escape speed = 1200 m s^{-1} ✓ (if correct equation is shown the previous mark is awarded without substitution)
So sufficient (initial) speed to escape. ✓

OR escape velocity = $\sqrt{\dfrac{2GM}{R}}$ substituting M from part (c) ✓
escape speed = 1200 m s^{-1} ✓ (1210 m s^{-1})
So sufficient (initial) speed to escape. ✓

OR escape velocity = $\sqrt{2Rg}$ substituting from data in (c) ✓

Third alternative may come from a CE from (c)

($1.06 \times 10^{-8} \times$ ($1.06 \times 10^{-8} \times \sqrt{\text{answer(c)}}$))

Conclusion must be explicit for third mark and cannot be awarded from a CE

3

[10]

12. **A**

[1]

13. **A**

[1]

14. **C**

[1]

15. (a) Total mass of spacecraft = 3050 kg

Change in PE = $\dfrac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 3050}{6400 \times 10^{3}}$

1.9×10^{11}(J)

2 sf

condone errors in powers of 10 and incorrect mass for payload

Allow if some sensible working

4

(b) Chemical combustion of propellant / fuel or gases produced at high pressure

Gas is expelled / expands through nozzle

Change in momentum of gases escaping

equal and opposite change in momentum of the spacecraft

Thrust = rate of change of change in momentum

Max 3

N3 in terms of forces worth 1

3

(c) 0.031(4) (m s^{-2})

1

(d) Use of rocket equation

$v = 1200 \ln \dfrac{3050}{1330}$

996 (m s^{-1})

Condone 1000 (m s^{-1})

3

(e) (i) Use of correct mass 108 kg

$F = \dfrac{6.67 \times 10^{-11} \times 1.1 \times 10^{13} \times 108}{(2 \times 10^{3})^{2}}$

0.0198 N

Allow incorrect powers of 10 and mass

3

(ii) Use of $v = \sqrt{\frac{2GM}{r}}$

Correct substitution $v = \frac{2 \times 6.67 \times 10^{-11} \times 1.1 \times 10^{18}}{2 \times 10^{8}}$

0.86 (m s^{-1})

Recognisable mass – condone incorrect power of 10

3

(iii) Impulse = 25 N × 4.8 = 120 N s

(120 = 108 v so) Velocity = 1.1 m s^{-1}

Clear conclusion

ie explanation/comparison of calculated velocity with escape velocity from **(e)(ii)**

May use F = ma approach

3

[20]

16. **C**

[1]

17. **C**

[1]

18. **B**

[1]