

A LEVEL PHYSICS

WORKED SOLUTIONS

6.1. Periodic Motion MCQ



1. A mass of 0.90 kg is suspended from the lower end of a light spring of stiffness 80 N m⁻¹.

When the mass is displaced vertically and released, it undergoes vertical oscillations of small amplitude.

What is the frequency of the oscillations?

$$T = 2\pi \sqrt{\frac{m}{k}} \quad f = \frac{1}{T}$$

- A 0.071 Hz
- B 0.67 Hz
- C 1.50 Hz
- D 14 Hz

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{80}{0.90}} = 1.501 \text{ Hz}$$

(Total 1 mark)

2. The period of a simple pendulum is doubled when the pendulum length is increased by 1.8 m.

What is the original length of the pendulum?

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \therefore T \propto \sqrt{L}$$

- A 0.45 m
- B 0.60 m
- C 0.90 m
- D 3.6 m

$$\frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}} \quad \frac{T_1^2}{T_2^2} = \frac{L_1}{L_2} \quad \frac{1}{4} = \frac{L_1}{L_1 + 1.8}$$

$$L_1 + 1.8 = 4L_1 \quad L_1 = 1.8/3 = 0.60 \text{ m}$$

(Total 1 mark)

3. A particle of mass m is oscillating with simple harmonic motion. The period of the oscillation is T and the amplitude is A .

What is the maximum kinetic energy of the particle?

$$E_{K \max} = \frac{1}{2} m v_{\max}^2$$

- A $\frac{mA^2}{2T^2}$
- B $\frac{\pi^2 mA^2}{2T^2}$
- C $\frac{2mA^2}{T^2}$
- D $\frac{2\pi^2 mA^2}{T^2}$

$$v_{\max} = \omega A = \frac{2\pi A}{T}$$

$$E_{K \max} = \frac{1}{2} m \frac{4\pi^2 A^2}{T^2} = \frac{2m\pi^2 A^2}{T^2}$$

(Total

4. A simple pendulum and a mass-spring system each have a time period T on the Earth. They are taken to the surface of a planet where the acceleration due to gravity is $\frac{g}{4}$. $T \propto \sqrt{\frac{1}{g}}$ ($\frac{1}{\sqrt{g}}$)

What are the time periods of the pendulum and the mass-spring system on this planet?

	Simple pendulum	Mass-spring system
A	$\frac{T}{2}$	T
<u>B</u>	$2T$	T
C	$\frac{T}{2}$	$2T$
D	$2T$	$2T$

$g \rightarrow g/4$

$\frac{1}{\sqrt{g/4}} = \frac{2}{\sqrt{g}}$

$\therefore 2T$

$\therefore 2T$

Mass-spring not affected (Total 1 mark)

5. A particle of mass m undergoes simple harmonic motion with amplitude A and frequency f . What is the total energy of the particle?

- A $2\pi m f A^2$
- B $2\pi^2 m f^2 A^2$
- C $4\pi^2 m^2 f^2 A$
- D $4\pi^2 m f^2 A^2$

Similar to Q3

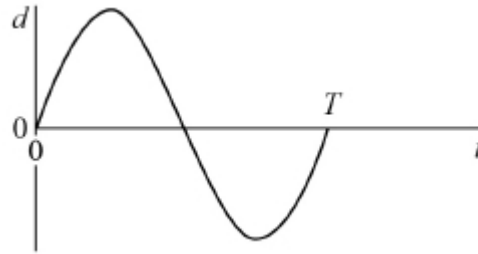
$E_T = E_{K_{max}} = \frac{1}{2} m v_{max}^2$

$v_{max} = \omega A = 2\pi f A$

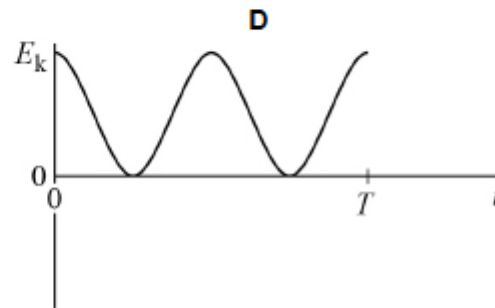
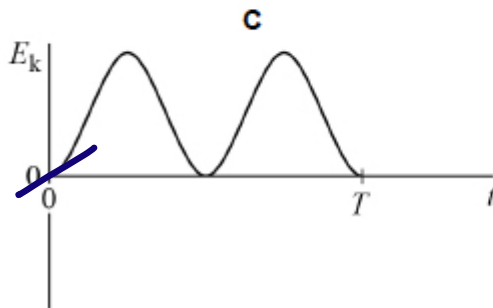
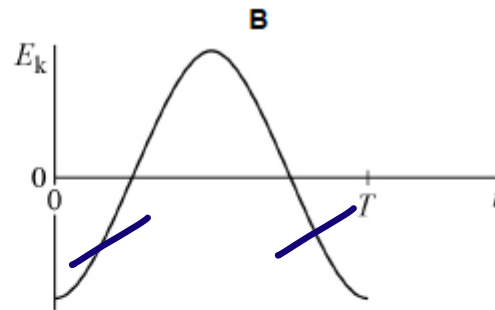
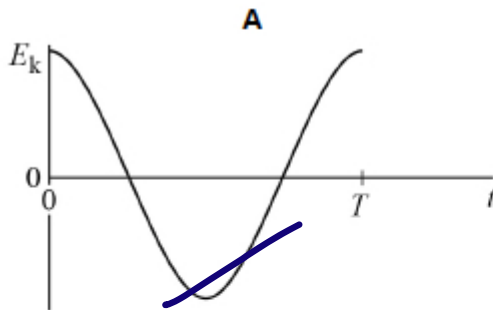
(Total 1 mark)

$$E_{K_{max}} = \frac{1}{2} m (2\pi f A)^2 = 2m\pi^2 f^2 A^2$$

6. The graph shows the variation of displacement d with time t for a particle moving with simple harmonic motion of period T .



Which graph shows the variation of kinetic energy E_k of the particle with time?



A

B

C

D

$$E_k \geq 0$$

at $t=0$ $d=0 \therefore v_{max}$ and E_{kmax}

(Total 1 mark)

7. Two pendulums **A** and **B** oscillate with simple harmonic motion. The time period of **A** is 2.00 s and the time period of **B** is 1.98 s.

A and **B** are released in phase.

What is the number of oscillations of **A** before **A** and **B** are next in phase?

A 49

B 50

C 99

D 100

$$\frac{1.98}{2.00 - 1.98} = \frac{1.98}{0.02} = 99$$

99 oscillations of A

(100 oscillations of B)

(Total 1 mark)

8. A helicopter circles continuously at a constant speed around a horizontal path of diameter 800 m, taking 5.0 minutes to complete each orbit of the path.

What are the speed v and the centripetal acceleration a of the helicopter?

	$v / \text{m s}^{-1}$	$a / \text{m s}^{-2}$	
A	0.021	0.18	<input type="radio"/>
B	8.4	0.088	<input type="radio"/>
<u>C</u>	8.4	0.18	<input checked="" type="radio"/>
D	17	0.35	<input type="radio"/>

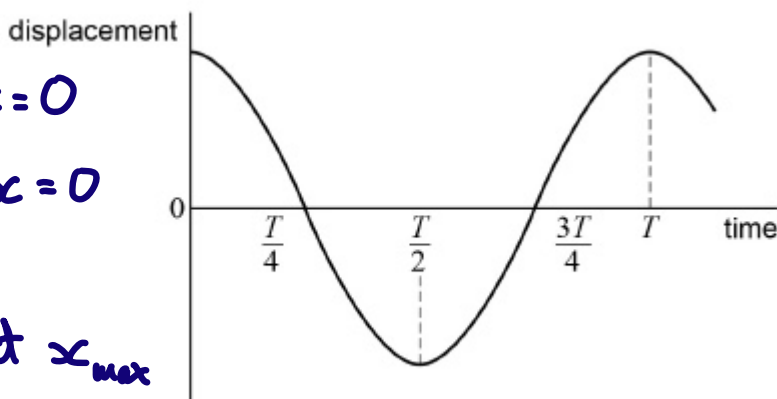
(Total 1 mark)

$$v = \frac{s}{t} = \frac{\pi d}{T} = \frac{800\pi}{300} = 8.38 \text{ m s}^{-1}$$

$$a = \frac{v^2}{r} = \frac{8.38^2}{400} = 0.175 \text{ m s}^{-2}$$

9. The graph shows how the displacement of a particle performing simple harmonic motion varies with time.

V_{max} at $x=0$
 $E_p=0$ at $x=0$
 $a \propto -x$
 $\therefore a_{max}$ at x_{max}

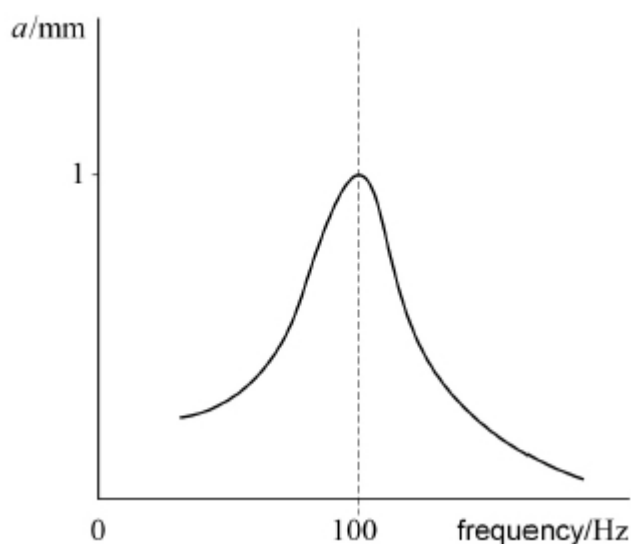


Which statement is **not** correct?

- A** The speed of the particle is a maximum at time $\frac{T}{4}$ ✓
B The potential energy of the particle is zero at time $\frac{3T}{4}$ ✓
C The acceleration of the particle is a maximum at time $\frac{T}{2}$ ✓
D The restoring force acting on the particle is zero at time T ✗

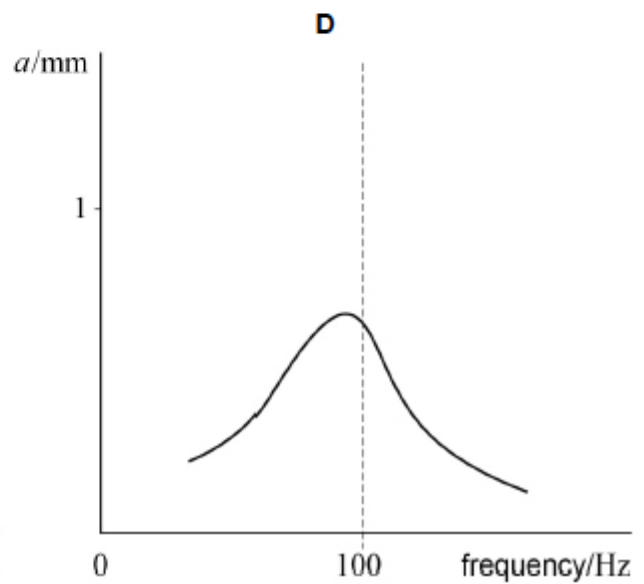
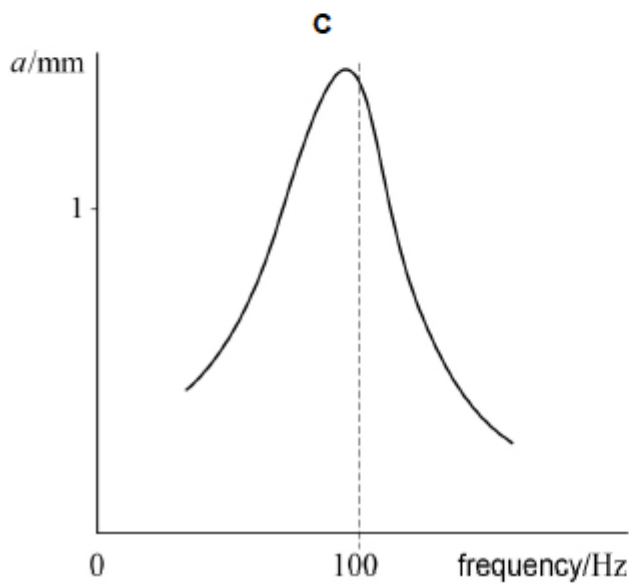
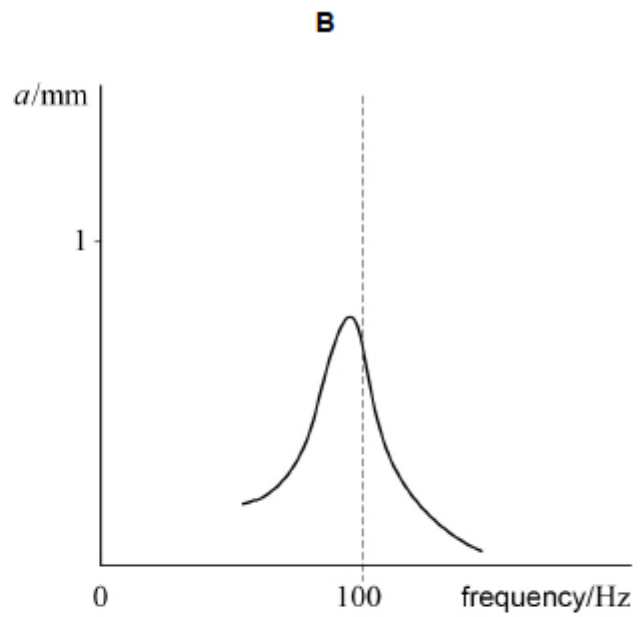
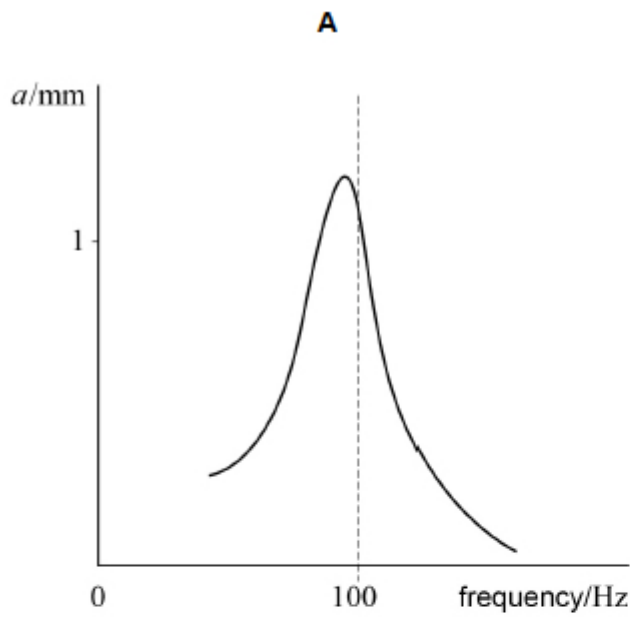
(Total 1 mark)

10. A metal panel is driven to vibrate at different frequencies. The amplitude a of the vibration is measured at each frequency. The graph shows the variation of amplitude with driven frequency.



The damping of the metal panel is increased without changing the mass of the panel.

Which graph shows the variation of a with frequency with increased damping?



A

B

C

D

Damped $\therefore a_{max} < 1$
Curve more spread out

(Total 1 mark)

11. The frequency of oscillation of a vertical spring is f when the mass hanging from the spring is m .

What is the relationship between f and m ?

A $f \propto m^{-1/2}$

B $f \propto m^{-2}$

C $f \propto m^{1/2}$

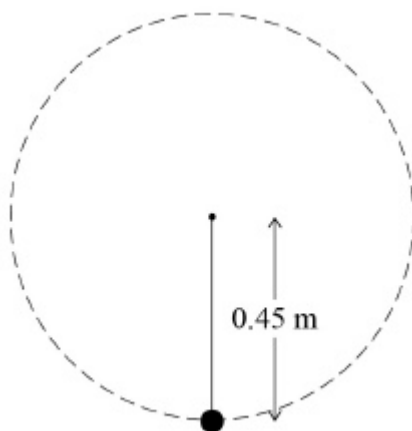
D $f \propto m^2$

$$T = 2\pi \sqrt{\frac{m}{k}} \quad f = \frac{1}{T}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad f \propto \sqrt{\frac{1}{m}}$$

(Total 1 mark)

12. A bob of mass 0.50 kg is suspended from the end of a piece of string 0.45 m long. The bob is rotated in a vertical circle at a constant rate of 120 revolutions per minute.



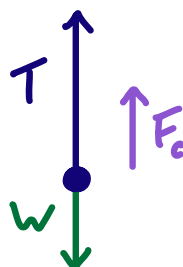
What is the tension in the string when the bob is at the bottom of the circle?

A 5.8 N

B 31 N

C 36 N

D 40 N



$$F_c = T - W$$

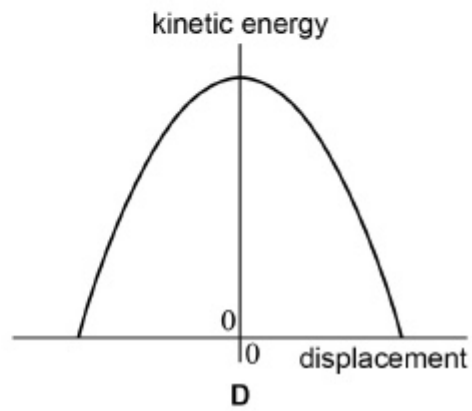
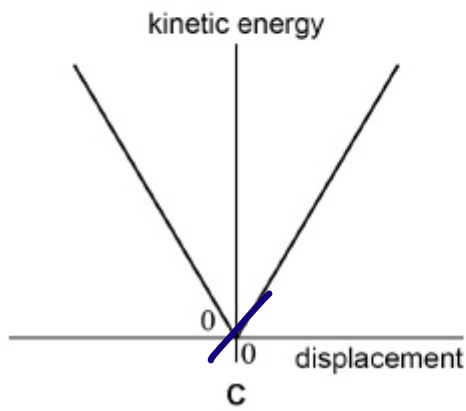
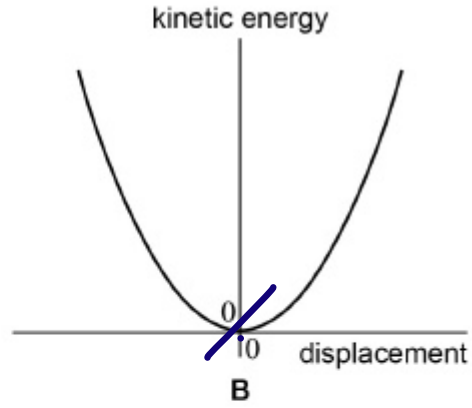
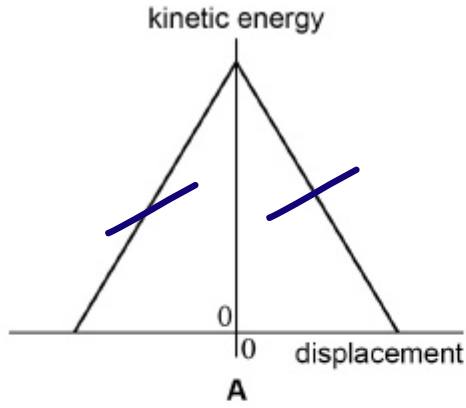
$$T = F_c + W$$

$$T = \frac{mv^2}{r} + mg$$

(Total 1 mark)

$$T = \frac{0.50 \times \left(\frac{120 \times \pi \times 0.45 \times 2}{60} \right)^2}{0.45} - 0.50 \times 9.81 = 40.4 \text{ N}$$

13. Which graph best shows how the kinetic energy of a simple pendulum varies with displacement from the equilibrium position?



A

B

C

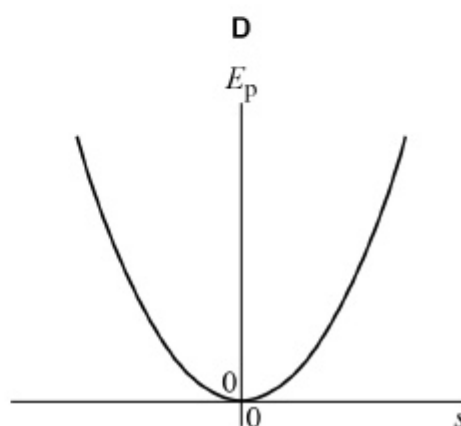
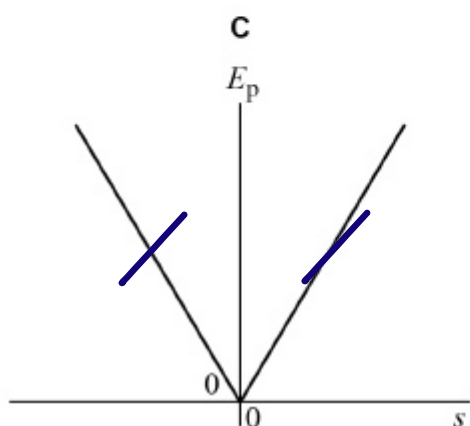
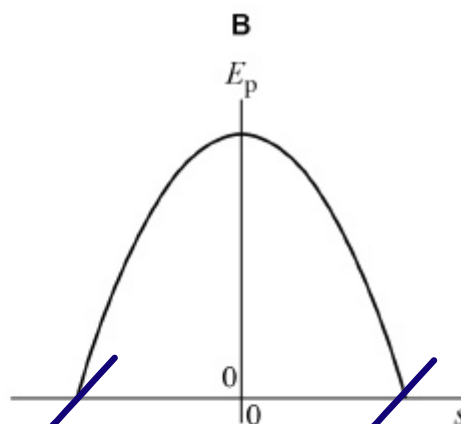
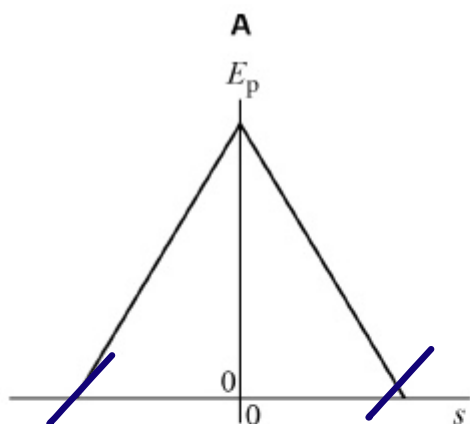
D

$$E_{K \max} \text{ at } x=0$$

$$E_k \propto v^2 \propto x^2$$

(Total 1 mark)

14. Which graph shows how the gravitational potential energy E_p of a simple pendulum varies with displacement s from the equilibrium position?



A

B

C

D

$$E_T = E_K + E_p$$

$$E_p = \text{constant} - E_K \quad (\text{opposite of Q13})$$

(Total 1 mark)

15. A body performs simple harmonic motion.

What is the phase difference between the variation of displacement with time and the variation of acceleration with time for the body?

- A 0
- B $\frac{\pi}{4}$ rad
- C $\frac{\pi}{2}$ rad
- D π rad

$a \propto -x$

Opposite directions

$\therefore 180^\circ$ phase difference

$\therefore \pi$ radians

(Total 1 mark)

16. An object of mass 0.15 kg performs simple harmonic motion. It oscillates with amplitude 55 mm and frequency 0.80 Hz

What is the maximum value of its kinetic energy?

- A 5.7×10^{-3} J
- B 11×10^{-3} J
- C 0.57 J
- D 11 J

$E_{K_{max}} = 2m\pi^2 f^2 A^2$
(from Q5)

$E_{K_{max}} = 2 \times 0.15 \times \pi^2 \times 0.80^2 \times 0.055^2$
 $= 0.00573$

(Total 1 mark)

17. An object of mass m moves in a circle of radius r . It completes n revolutions every second.

What is the kinetic energy of the object?

- A $\frac{mn^2 r^2}{8\pi^2}$
- B $\frac{mn^2 r^2}{4\pi^2}$
- C $2m\pi^2 n^2 r^2$
- D $4m\pi^2 n^2 r^2$

$E_k = \frac{1}{2}mv^2$

$E_k = \frac{1}{2}m \left(\frac{2\pi n r}{t} \right)^2$

$t = 1$

$E_k = 2m\pi^2 n^2 r^2$

(Total 1 mark)