1. The Global Positioning System (GPS) uses satellites to support navigation on Earth.

(a) One GPS satellite is in a circular orbit at a height h above the surface of the Earth. The Earth has mass M and radius R.

Show that the angular speed of the satellite is given by

$$\omega = \sqrt{\frac{GM}{(R+h)^3}}$$

(2)

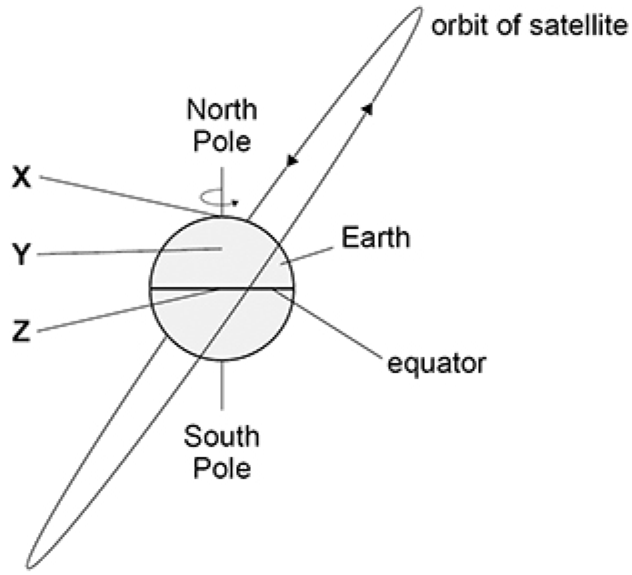
(b) Calculate the orbital period of the satellite when h equals 2.02×10^7 m.

orbital period = ________________ s

(2)

(c) The figure below shows the orbital plane of the satellite inclined at an angle to the equator. **X**, **Y** and **Z** are locations on the Earth.

X is at the North Pole, **Y** is on a high mountain and **Z** is on the equator.



The satellite is to be launched from one of the locations.

State and explain which launch site **X**, **Y** or **Z** minimises the amount of fuel required to send the satellite into its orbit.

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(2)

(d) The satellite has a mass of 1630 kg.

Calculate the gravitational potential energy of the satellite when in the orbit in part (b).

gravitational potential energy = _______________ J

(2)

(e) A different satellite is in a higher circular orbit.

Explain how the linear speed of this satellite compares with the linear speed of the satellite in part (a).

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(2)

(Total 10 marks)

2. The distance between the Sun and the Earth is 1.5×10^{11} m

What is the gravitational force exerted on the Sun by the Earth?

A 3.5×10^{22} N

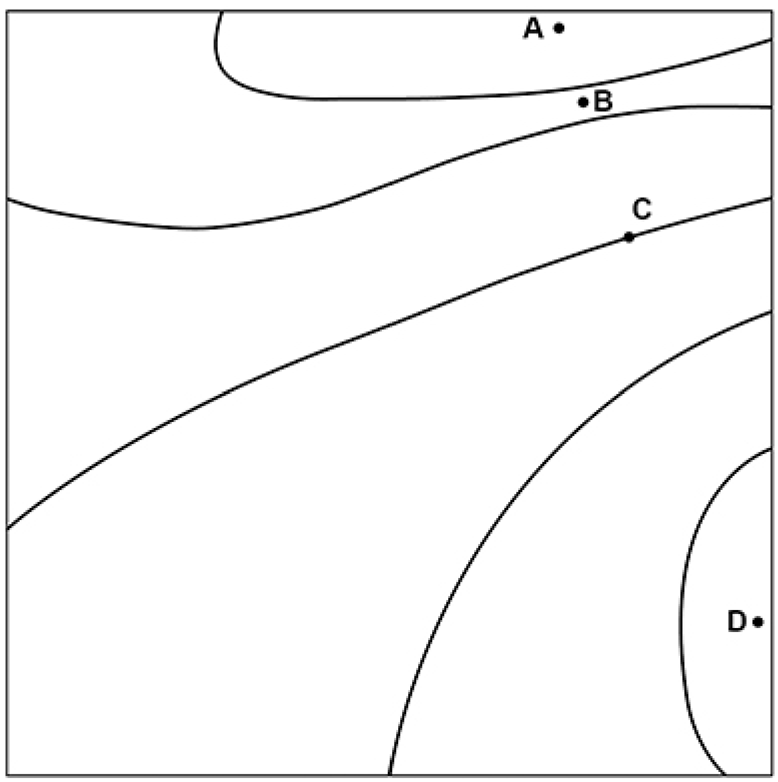
B 1.7×10^{26} N

C 5.3×10^{33} N

D 8.9×10^{50} N

(Total 1 mark)

3. The diagram shows gravitational equipotentials. Adjacent equipotentials are separated by an equal gravitational potential difference V.



Which point has the greatest gravitational field strength?

A

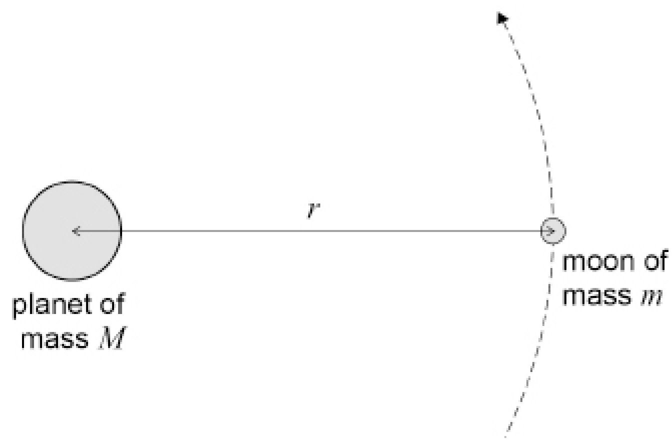
B

C

D

(Total 1 mark)

4. The figure shows a moon of mass m in a circular orbit of radius r around a planet of mass M, where $m << M$.



The moon has an orbital period T.
T is related to r by

$$T^2 = kr^3$$

where k is a constant for this planet.

(a) Show that $k = \dfrac{4\pi^2}{GM}$

(3)

Table 1 gives data for two of the moons of the planet Uranus.

Table 1

Name	***T* / days**	***r* / m**
Miranda	1.41	1.29×10^8
Umbriel	4.14	**X**

(b) Calculate the orbital radius **X** of Umbriel.

orbital radius = ________________ m

(2)

(c) Calculate the mass of Uranus.

mass = ________________ kg

(3)

Table 2 gives data for three more moons of Uranus.

Table 2

Name	**Mass / kg**	**Diameter / m**
Ariel	1.27×10^{21}	1.16×10^{6}
Oberon	3.03×10^{21}	1.52×10^{6}
Titania	3.49×10^{21}	1.58×10^{6}

(d) Deduce which moon in **Table 2** has the greatest escape velocity for an object on its surface.
Assume the effect of Uranus is negligible.

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(3)

(e) A spring mechanism can project an object vertically to a maximum height of 1.0 m from the surface of the Earth.

Determine whether the same mechanism could project the same object vertically to a maximum height greater than 100 m when placed on the surface of Ariel.

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(3)
(Total 14 marks)

5. A planet has radius R and density ρ. The gravitational field strength at the surface is g.

What is the gravitational field strength at the surface of a planet of radius $2R$ and density 2ρ?

A $2g$ ☐

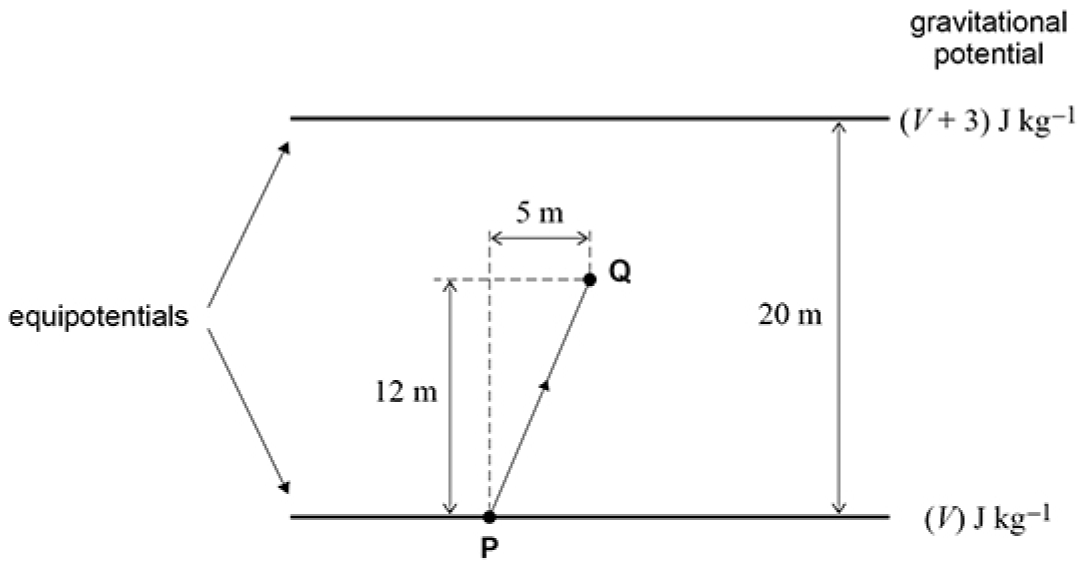
B $4g$ ☐

C $8g$ ☐

D $16g$ ☐

(Total 1 mark)

6. The diagram shows equipotential lines for a uniform gravitational field. The lines are separated by 20 m.



An object of mass 4 kg is moved from **P** to **Q**.

What is the work done against gravity to move the object?

A 7.2 J ○

B 7.8 J ○

C 10.2 J ○

D 36 J ○

(Total 1 mark)

7. (a) Define gravitational potential at a point.

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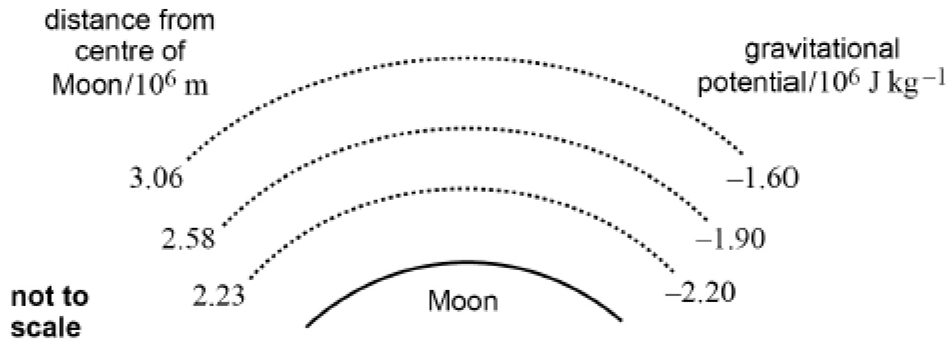
(1)

(b) The diagram shows the positions of equipotential surfaces at different distances from the centre of the Moon.



Explain how the equipotential surfaces in the diagram show that the gravitational field is **not** uniform.

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(1)

(c) Calculate, using the diagram above, the escape velocity at the surface of the Moon.

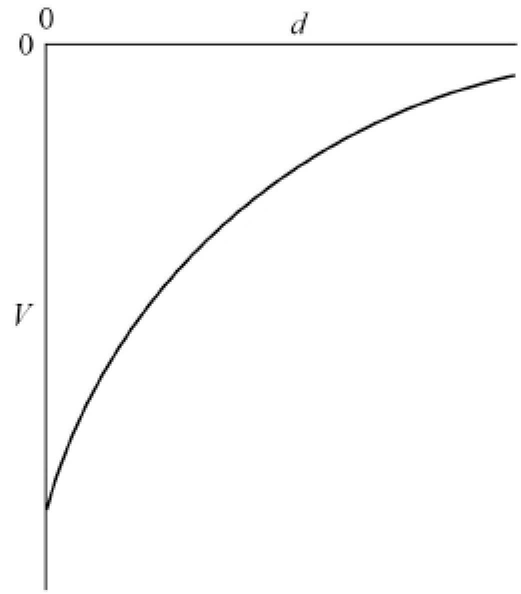
radius of Moon = 1.74×10^6 m

escape velocity = ____________________ m s^{-1}

(4)

(Total 6 marks)

8. The graph shows how the gravitational potential V varies with the vertical distance d from the surface of the Earth.



What does the gradient of the graph represent at the surface of the Earth?

A potential energy

B mass of the Earth

C magnitude of the gravitational constant

D magnitude of the gravitational field strength

(Total 1 mark)

9. What is the angular speed of a satellite in a geostationary orbit around the Earth?

A 1.2×10^{-5} rad s^{-1}

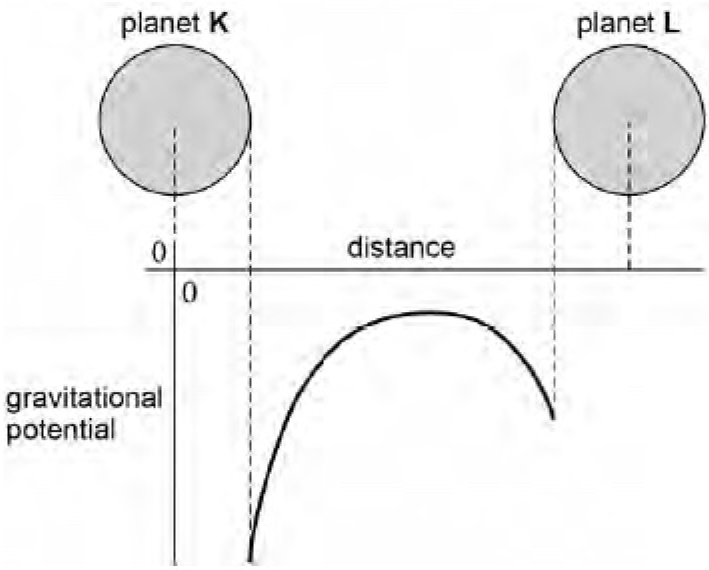
B 7.3×10^{-5} rad s^{-1}

C 4.4×10^{-3} rad s^{-1}

D 2.6×10^{-1} rad s^{-1}

(Total 1 mark)

10. The graph shows how the gravitational potential varies with distance between two planets, **K** and **L**, that have the same radius.



Which statement is correct?

A The mass of **L** is greater than the mass of **K**. ⬭

B The gravitational field strength at the surface of **L** is greater than that at the surface of **K**. ⬭

C The escape velocity from planet **L** is greater than that from planet **K**. ⬭

D More work must be done to move a mass of 1 kg from the surface of **K** to a distant point, than 1 kg from the surface of **L**. ⬭

(Total 1 mark)

11. (a) Derive an expression to show that for satellites in a circular orbit

$$T^2 \propto r^3$$

where T is the period of orbit and r is the radius of the orbit.

(2)

(b) Pluto is a dwarf planet. The mean orbital radius of Pluto around the Sun is 5.91×10^9 km compared to a mean orbital radius of 1.50×10^8 km for the Earth.

Calculate in years the orbital period of Pluto.

orbital period of Pluto = ____________________ yr

(2)

(c) A small mass released from rest just above the surface of Pluto has an acceleration of 0.617 m s^{-2}.

Assume Pluto has no atmosphere that could provide any resistance to motion.

Calculate the mass of Pluto.

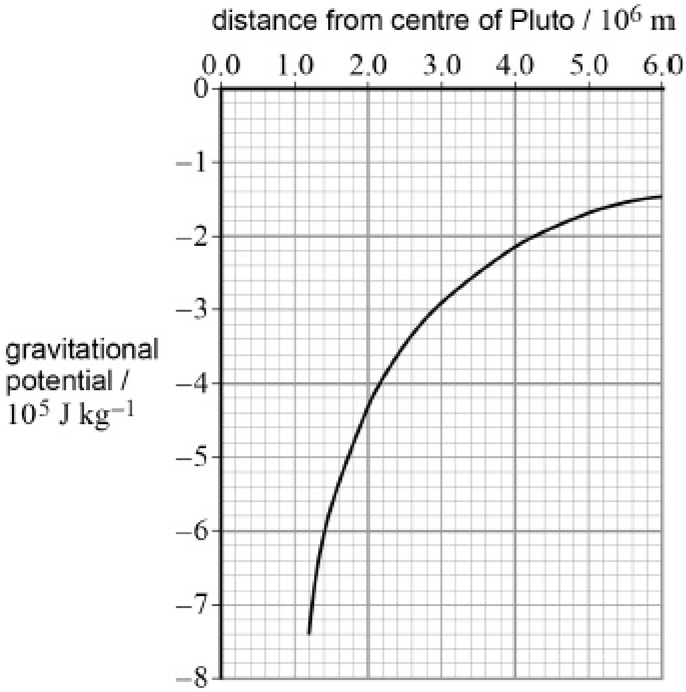
Give your answer to an appropriate number of significant figures.

radius of Pluto = 1.19×10^6 m

mass of Pluto = ____________________ kg

(3)

(d) The graph shows the variation in gravitational potential with distance from the centre of Pluto for points at and above its surface.



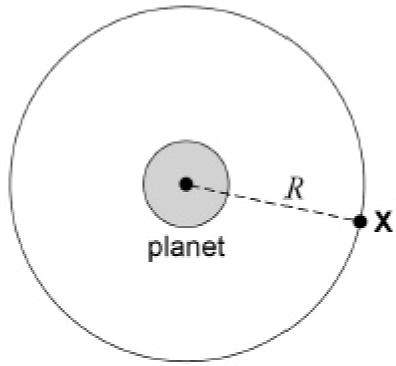
A meteorite hits Pluto and ejects a lump of ice from the surface that travels vertically at an initial speed of 1400 m s^{-1}.

Determine whether this lump of ice can escape from Pluto.

(3)

(Total 10 marks)

12. A satellite **X** of mass m is in a concentric circular orbit of radius R about a planet of mass M.



What is the kinetic energy of **X**?

A $\frac{GMm}{2R}$ ☐

B $\frac{GMm}{R}$ ☐

C $\frac{2GMm}{R}$ ☐

D $\frac{4GMm}{R}$ ☐

(Total 1 mark)

13. The distance between the Sun and Mars varies from 2.1×10^{11} m to 2.5×10^{11} m.
When Mars is closest to the Sun, the force of gravitational attraction between them is F.

What is the force of gravitational attraction between them when they are furthest apart?

A $0.71F$ ☐

B $0.84F$ ☐

C $1.2F$ ☐

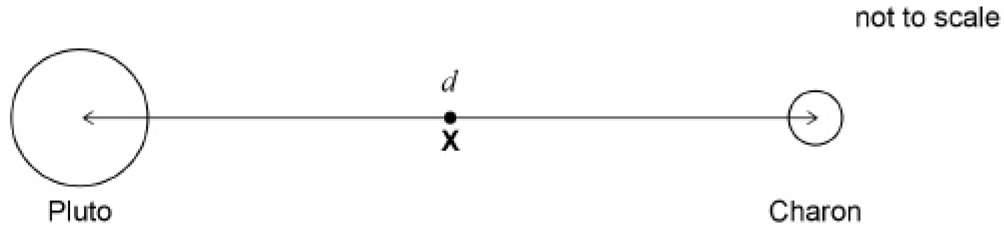
D $1.4F$ ☐

(Total 1 mark)

14. Charon is a moon of Pluto that has a mass equal to $\frac{1}{9}$ that of Pluto.

The distance between the centre of Pluto and the centre of Charon is *d*.

X is the point at which the resultant gravitational field due to Pluto and Charon is zero.



What is the distance of **X** from the centre of Pluto?

A $\frac{2}{9}d$ ☐

B $\frac{2}{3}d$ ☐

C $\frac{3}{4}d$ ☐

D $\frac{8}{9}d$ ☐

(Total 1 mark)

15. The Rosetta space mission placed a robotic probe on Comet 67P in 2014.

(a) The total mass of the Rosetta spacecraft was 3050 kg. This included the robotic probe of mass 108 kg and 1720 kg of propellant. The propellant was used for changing velocity while travelling in deep space where the gravitational field strength is negligible.

Calculate the change in gravitational potential energy of the Rosetta spacecraft from launch until it was in deep space.
Give your answer to an appropriate number of significant figures.

Mass of the Earth = 6.0×10^{24} kg
Radius of the Earth = 6400 km

change in gravitational potential energy ____________________ J

(4)

(b) As it approached the comet, the speed of the Rosetta spacecraft was reduced to match that of the comet. This was done in stages using four 'thrusters'. These were fired simultaneously in the same direction.

Explain how the propellant produces the thrust.

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(3)

(c) Each thruster provided a constant thrust of 11 N.

Calculate the deceleration of the Rosetta spacecraft produced by the four thrusters when its mass was 1400 kg.

deceleration ____________________ m s^{-2}

(1)

(d) Calculate the maximum change in speed that could be produced using the 1720 kg of propellants.

Assume that the speed of the exhaust gases produced by the propellant was 1200 m s^{-1}

maximum change in speed ____________________ m s^{-1}

(3)

(e) When the robotic probe landed, it had to be anchored to the comet due to the low gravitational force. Comet 67P has a mass of about 1.1×10^{13} kg. A possible landing site was about 2.0 km from the centre of mass.

(i) Calculate the gravitational force acting on the robotic probe when at a distance of 2.0 km from the centre of mass of the comet.

gravitational force ____________________ N

(3)

(ii) Calculate the escape velocity for an object 2.0 km from the centre of mass of the comet.

escape velocity ____________________ m s^{-1}

(3)

(iii) A scientist suggests using a drill to make a vertical hole in a rock on the surface of the comet. The anchoring would be removed from the robotic probe before the drill was used. The drill would exert a force of 25 N for 4.8 s.

Explain, with the aid of a calculation, whether this process would cause the robotic probe to escape from the comet.

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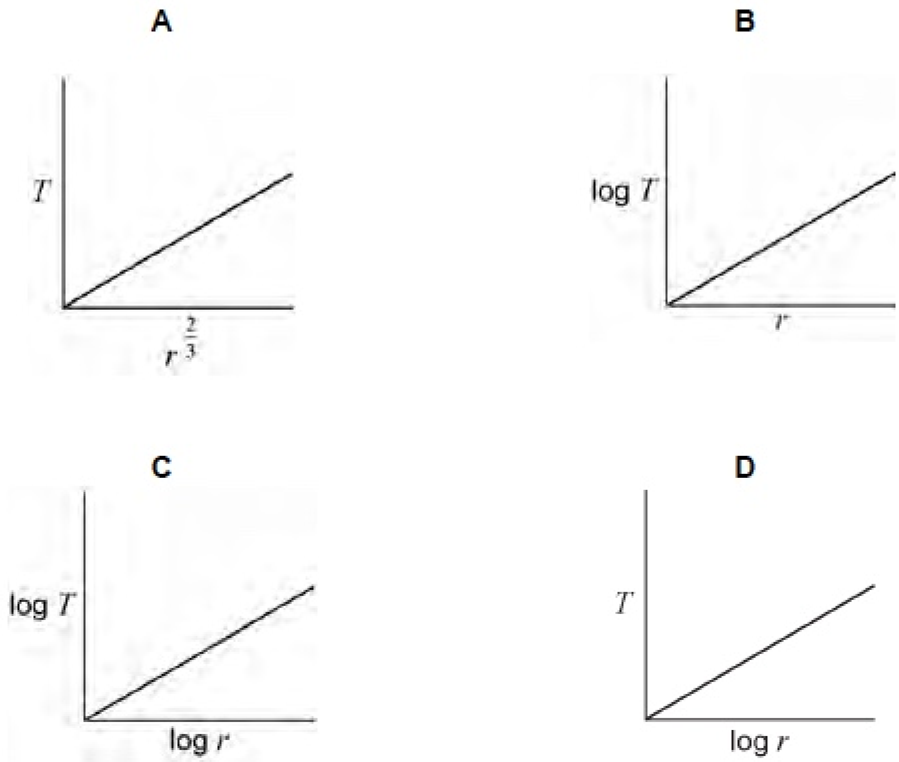
(3)

(Total 20 marks)

16. Which graph shows the relationship between the time period T and the orbital radius r of a planet in orbit around the Sun?



A ⬭

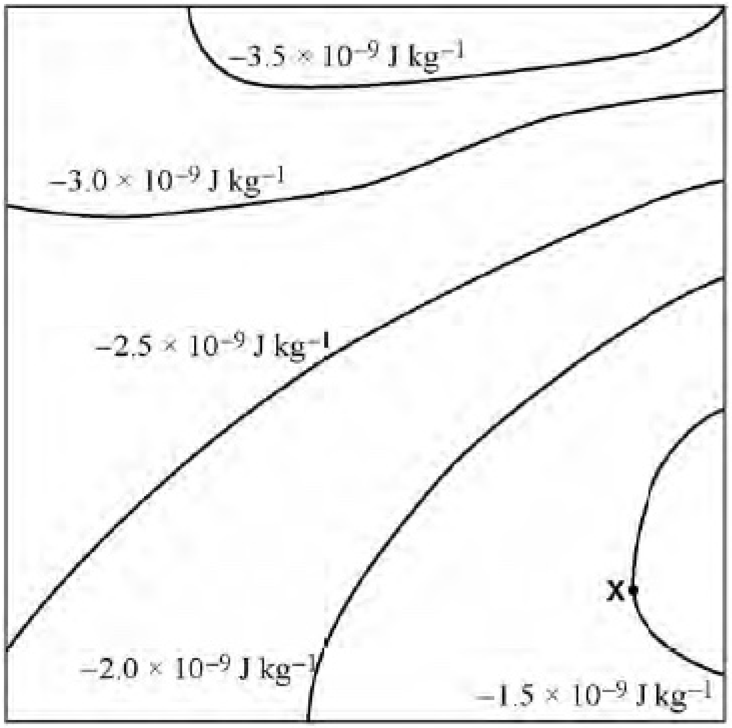
B ⬭

C ⬭

D ⬭

(Total 1 mark)

17. The diagram shows equipotential lines near a group of asteroids.



Which arrow shows the direction of the gravitational field at **X**?

A $\uparrow$

B $\downarrow$

C $\leftarrow$

D $\rightarrow$

(Total 1 mark)

18. Planet **N** has a gravitational potential $-V$ at its surface. Planet **M** has double the density and double the radius of planet **N**. Both planets are spherical and have uniform density.

What is the gravitational potential at the surface of planet **M**?

A $-16V$

B $-8V$

C $-4V$

D $-0.2V$

(Total :