Mark schemes

1. (a) (The M of I decreases) because more mass closer to axis of rotation ✓$_1$

$I\omega$/angular momentum constant since no external torque ✓$_2$

since I decreases, ω must increase ✓$_3$

For ✓$_1$ must have the idea of mass distribution around axis of rotation. Do not accept answers which give only decrease in radius as reason for lower M of I.

For ✓$_2$ condone answers which do not mention the condition of no external torque. ✓$_3$ cannot be awarded if conservation of rotational kinetic energy used.

3

(b) $I_1\omega_1 = I_2\omega_2$ $\omega_1 = 4.3$ rad s^{-1} ✓

Accept the answer 4.3 rad s^{-1} if no working shown.

1

(c) Finds time for one rotation ✓

Divides 1.2 by time

AND

gives answer for complete rotations, not rounded up. ✓

time for 1 rotation = 2π/14.2 = 0.442 s

1.2/0.442 = 2.7 rotations/turns/somersaults.

OR Angle turned through = 14.2 × 1.2 = 17.04 rad

17.04/2π = 2.7 rotations

OR

Finds angle turned through in 1.2 s ✓

Divides by 2π

AND

gives answer for complete rotations, not rounded up. ✓

Expect to see 2 complete rotations/turns/somersaults.

For MP2 give CE for time or angle from MP1

2

(d) Any 2 from:

- build up a greater initial angular speed around the bar ✓
  so reaches a greater height/will rotate faster in tuck ✓
- release at a greater angle from the horizontal ✓
  so will rise to greater height giving more time for somersaults ✓
- get into tuck position earlier/get out of tuck position later ✓
  so turning for more time ✓
- get into tighter tuck position ✓ reducing I_2, and increasing ω_2 ✓

Any 2

statement ✓ and correct reason ✓ scores 2 marks for each.

4

[10]

2. (a) Equates initial E_p to linear E_k and rotational E_k ✓
Substitutes values and uses $V = r\omega$ ✓
Calculates V to give 0.51 m s^{-1} ✓

$9.2 \times 10^{-2} \times 9.81 \times 0.5 = (\frac{1}{2} \times 9.2 \times 10^{-2}\, V^2) + (\frac{1}{2} \times 8.6 \times 10^{-5} \times \frac{V^2}{0.005^2})$

$V = 0.51$ m s^{-1}

Some substitution of data must be seen for MP2

Do not allow MP3 for no consideration of linear E_k

Give 1 mark if mgh = $\frac{1}{2} I\omega^2$ used with answer 0.51 m s^{-1}

3

(b) Calculates α from $\alpha = T/I$ ✓

Attempts to use any appropriate equation(s) of motion (for angular motion) ✓

Substitutes into equation(s) of motion and calculates θ ✓

$\alpha = (8.3 \times 10^{-4}) / 8.6 \times 10^{-5} = 9.65$ rad s^{-2}

or 9.7 rad s^{-2}

$\theta = 145 \times 10 - \frac{1}{2} \times 9.7 \times 10^2 = 967$ rad or 970 rad

MP2: $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ is not enough on its own as there are two unknowns.

MP2: Quoting appropriate formula(e) is not enough. There must be some attempt at substituting the data.

3

[6]

3. (a) No (net) external torque acts (on the system) ✓

Do not accept force for torque

1

(b) $I_A \omega_A + I_B \omega_B = (I_A + I_B)\,\omega$ ✓
(taking clockwise as positive)
$(7.2 \times 95) + (11.5 \times -45) = 18.7\omega$
ω = (+)8.9 rad s^{-1} ✓
clockwise ✓

Accept answers with anticlockwise taken as positive.

1st mark for equation or substitution, but condone any incorrect sign for angular velocity.

2nd mark: answer to at least 2 sf

3rd mark for direction, ECF provided direction agrees with sign in calculated answer and sign convention used.

3rd mark is not an independent mark and is contingent on some attempt at calculation using angular momentum

3

(c) Attempts to use Angular impulse $= Tt = \Delta(I\,\omega)$ ✓
Clutch C: 600 t = 7.2 × (95 − 8.9) = 620 (N m s)
t = 1.03 s
OR α = (95 − 8.9)/t
600 = $I\,\alpha$ = 7.2 (95 − 8.9)/t
t = 1.03 s
Clutch D: 320 t = 7.2 × (95 − 8.9)
t = 1.93 s ✓ (for either or both times calculated)

Compares correct times with 1s < t < 2s and concludes both clutches satisfy criterion. ✓

1st mark: attempts to use idea of angular impulse

Mark not given for just quoting formula.

2nd mark: correct time(s) calculated for either or both clutches

OR torques calculated for 1 s and/or 2 s [620 Nm and 310 Nm]

3rd mark: correct conclusion based on correct times for both clutches

OR based on comparing calculated torques for 1 and 2 s with data in Table 2

Answers may be worked out using shaft B:

$T \times t = 11.5 \times (-45 - 8.9) = (-)620$ N m s

Give full marks if 9 rad s^{-1} is used, giving

angular impulse = 619 N m s

t for clutch C = 1.03 s

t for clutch D = 1.93 s

3

[7]

4. (a) Work done ✓

1

(b) **The mark scheme gives some guidance as to what statements are expected to be seen in a 1 or 2 mark (L1), 3 or 4 mark (L2) and 5 or 6 mark (L3) answer. Guidance provided in section 3.10 of the 'Mark Scheme Instructions' document should be used to assist marking this question.**

Mark	Criteria
6	There is a response to all 3 bullet points in the question. There is a good understanding of the function of a flywheel, and why the torque varies markedly in a diesel engine. Student can relate the answer to the two graphs. Includes 6 or more answer points from the list alongside
5	There is a response to all 3 bullet points in the question covering 6 answer points. Answers will not be as confident or detailed as for 6 marks, or answers may not be expressed using scientific terminology.
4	The student gives five or more answer points covering at least two of the bullet points.
3	At least four pertinent statements. They may show little understanding of the electric motor but should be able to give some reasons why a diesel engine needs a flywheel.
2	Two or three pertinent statements taken from the list of likely answer points.
1	One pertinent statement.
0	No sensible statements made.

Other sensible and applicable points can be accepted in lieu of any of those alongside.

Likely answer points:

<u>1st bullet</u>

1. *Electric motor's constant torque means smooth motion/doesn't need smoothing/doesn't need a flywheel*
2. *motor's output torque matches the described load*

<u>2nd bullet</u>

3. *relates force/pressure on piston to torque*
4. *force on piston varies over one cycle (as pressure in cylinder varies)*
5. *Torque = Fr and effective r varies as crank rotates*
6. *-ve torque: when work is being done on (the gas in) the engine (during induction, comp, exhaust strokes)*
7. *Zero torque when con rod and crank are in line/at top and bottom dead centres*
8. *This happens at crank angles which are multiples of π*

<u>3rd bullet</u>

9. *Diesel engine's (varying torque) will give uneven/jerky motion/cause stalling*
10. *Flywheel acts as energy store*
11. *Flywheel absorbs energy on power/expansion stroke*
12. *and gives up energy on other parts of cycle*
13. *Flywheel speeds up on expansion stroke*
14. *and slows down during other strokes.*
15. *The greater the M of I of flywheel, the smoother the motion*
16. *If no flywheel engine will stall/become very uneven/jerky*
17. *The greater the M of I of flywheel, the longer engine will take to speed up, slow down/stop*
18. *Because machine has low M of I it will not be able to store energy itself or smooth the motion.*

6

[7]

5. (a) Sum of all constituent masses × their radius/distance from the axis squared

Allow Σmr^2 with m defined as small mass or constituent mass or particle at a radius r and Σ explained.

Condone: 'from the axis' missing

Condone: a quantity expressing a body's tendency to resist angular acceleration/change in angular speed

1

(b) E_P lost (by falling mass) = E_K pulley + E_K mass ✓

$0.5Mgh = ½(0.5M)v^2 + ½(0.5MR^2)\,\omega^2$
Cancel 0.5 and M and substitute $\omega = {}^{v}\!/_{R}$ for ω
gives $gh = ½v^2 + ½v^2 = v^2$ ✓
use of $v^2 = u^2 + 2as$ giving $v^2 = 2ah$ ✓
substitutes $v^2 = 2ah$ in $gh = v^2$ (so $a = 0.5g$) ✓

*1st mark for equating E_P lost by mass to E_K gained by **both** mass and pulley. Accept this step in words or symbols*

2nd mark for $gh = v^2$

3rd mark for v^2 in terms of h

4th mark for combining correctly (to get $a = 0.5g$)

OR
$0.5Mg - F = 0.5Ma$ ✓
Torque = $I\alpha$ $F \times R = (0.5MR^2)\alpha$ ✓
(giving $F = 0.5\ MR\alpha$)
and substitute $\alpha = a/R$
leading to $F = 0.5Ma$ ✓
Substitute for F in $0.5Mg - F = 0.5Ma$ (gives $a = 0.5g$) ✓

OR with F or other letter as tension in string:

1st mark for Newton's 2nd law applied to mass in words or symbols

2nd mark for accelerating torque equation

3rd mark F in terms of a

4th mark for substituting to get $a = 0.5g$

Note: $\alpha = a/R$ is not in the spec, but students may know it and use this route.

Give ECF if M is used for the falling mass in place of $0.5M$

4

(c)

Route 1	Route 2
M of I spoked pulley is greater ✓1 Reason given for greater M of I but must have reference to distribution or spread of mass about axis ✓2 Greater proportion of E_P loss given to pulley OR lower prop to E_K of falling mass ✓3 v of mass in same time will be lower so acceleration lower ✓4	M of I spoked pulley is greater ✓1 Reason given for greater M of I but must have reference to distribution or spread of mass about axis ✓2 Presents valid argument relating I to α ✓3 $\alpha = a/R$ (with α less) so acceleration of mass is less OR wheel turns through fewer rotations in same time so point on rim moves less distance so acceleration less ✓4

WTTE

For ✓3 and ✓4 marks in route 2

$0.5M(g - a) = F$

$0.5M(g - a)R = I\alpha$

$0.5Mg = a\,(0.5M + I/R^2)\,a$

If I increases, a decreases.

Max 3

[7]

6. (a) The (total) <u>angular</u> momentum (of a system) remains constant provided no external <u>torque</u> acts (on the system) ✓

Must see 'angular'. Condone 'is conserved' for 'is constant'

Allow ang momtm before equals/is same as ang momtm after OR initial ang momtm = final ang momtm

Allow $I\omega$ is constant if symbols explained

Do not allow 'force' in place of 'torque'

1

(b) Use of $I = I_{BODY} + 2 \times mr^2$ ✓

$I_1 = (71 + 2 \times 5.0 \times 4.1^2) = 239$ kg m^2 ✓

(≈ 240 kg m^2)

For 2 marks 239 must be seen

2

(c) M of I decreases ✓

Because more mass closer to axis **OR** (for pods) $\underline{I = (\sum)mr^2}$ with r less ✓

$I\omega$ / angular momentum remains constant/is conserved

(So as I decreases) ω must increase ✓

Condone 'inertia' for 'moment of inertia'

2nd mark is for the reason why I is decreasing. Answer must relate to pods or masses getting closer to the axis. 'radius decreasing' on its own is not enough. Accept: pods get closer to axis/body as this implies mass is getting closer.

Both points needed for 3rd mark

3

(d) (Applies conservation of angular momentum/ $I_1\omega_1 = I_2\omega_2$)

$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)})$ ✓

$312 = (71 + 2 \times 5.0 \times 0.74^2)\ \omega_2$

$\omega_2 = 4.08 \text{ rad s}^{-1}$ ✓

Therefore max speed not reached OR arms can be retracted safely✓

OR

$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)})$ ✓

$312 = (71 + 2 \times 5.0 \times r_2^2)\ 4.2$

$r_2 = 0.57$ m ✓

So with r at circumference max speed not reached OR arms can be retracted safely✓

OR

$I_1\omega_1 = 240 \times 1.3 = (312 \text{ (N m s)})$ ✓

$312 = 4.2\, I_2$ at safety limit

$I_2 = 74(.3)$ kg m^2 ✓

Actual $I_2 = 76.5$ kg m^2

Therefore max speed not reached OR arms can be retracted safely✓

Using 239 kg m^2 instead of 240 kg m^2 leads to

$\omega^2 = 4.06$ rad s^{-1}

Useful: $I_2 = 76.5$ kg m^2

Only credit last mark if conservation of angular momentum is used

Allow a judgement based on incorrect working (eg AE) provided conservation of angular momentum is used

Using 239 kg m^2 instead of 240 kg m^2 leads to

$r_2 = 0.55$ m

3

[9]

7. (a) 2.9 rev s^{-1} equivalent = $2\pi \times 2.9$ rad s^{-1} = 8.2 rad s^{-1}

OR $I = 2\, E_k / \omega^2$

OR correct substitution in $E_k = ½\, I\, \omega^2$ ✓

leading to $I = 6.2 \times 10^{-2}$ kg m^2 ✓

1st mark for correct conversion rev s^{-1} OR rearranging energy equation in terms of I OR correct substitution in $E_k = ½\, I\, \omega^2$

2nd mark for correct answer.

Do not allow final answer to 1 sig fig e.g. 0.06

2

(b) I depends on how mass is distributed about axis (of rotation)

For arms, screw and punch same mass is/point masses are closer to axis than the steel balls (making M of I lower) ✓

I depends on r^2 so I changes greatly for small change in in r ✓

Allow ‘other parts’ or ‘other components’ if it is clear this means screw, punch and arms

2

(c) $\alpha = \frac{2 \times \pi \times (0 - 2.9)}{0.089} = -205 \text{ rad s}^{-2}$ ✓

Attempt to use $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ or $\theta = \omega 1t + ½ \alpha t^2$

or $\theta = ½ (\omega_1 + \omega_2)t$ ✓

giving $\theta = 0.81$ rad ✓

Condone missing sign or α given as positive Accept 200 rad s^{-2}

If α positive, 2nd mark for attempt to use

$\omega_2^2 = \omega_1^2 - 2\alpha\theta$ or $\theta = \omega_1 t - ½ \alpha t^2$

or $\theta = ½(\omega_1 + \omega_2)t$ ✓

ECF for value of ω used in (a)

3

(d) ($I = 2\ mr^2$ and $E_k = ½\ I\ \omega 2$)

Increasing y by 15% gives new $I = 1.15^2 \times$ original I (or 1.32) ✓

Increasing R by 15% increases I by 1.15^3 (or 1.52) ✓

Second option gives greater increase in I, and E_k also increased (by same ratio). ✓

Accept answers without calculation:

I prop to y^2 ✓

I prop to R^3 ✓

For <u>same</u> % increase in y or R, I and hence E_k increases more by increasing R ✓

Note: $E_k = m\ r^2\ \omega^2 = 4/3\ \pi\ R^3\ \rho\ r^2\ \omega^2$ for each ball

3

(e) ✓ against N m s

1

[11]

8. (a)

Translational dynamics	**Rotational dynamics**
force	torque ✓
mass	moment of inertia ✓

Do not allow 'inertia'

2

(b) $I_T = 2.6 \times 10^7 + (2.2 \times 10^3 \times 35^2) = 2.9 \times 10^7$ (kg m^2) ✓

Mark only awarded for arriving at correct answer to more than 1 sf.

1

(c) Use of (total) area under graph = (angular) displacement/distance ✓

$\omega_{max}((½ \times 30) + 20 + (½ \times 45)) = 4.7$

$\omega_{max} = 0.082$ (rad s-1) ✓

Alternative route is area of trapezium

½ ω_{max} (20 + 95) = 4.7

2

(d) moment of inertia of rotating jib + load increases as trolley moves outwards ✓

reference to $T = I\alpha$ with T constant, so α decreases ✓

decreased α means longer time to stop(than 95 s) ✓

3

[8]

9. (a) $T = 6.0 \times 0.036 = 0.22$ (N m) ✓

1

(b) power cannot increase ✓

$P = T\omega$ so if ω is 4x greater, T cannot be more than 1/4✓

OR

Work done by (torque) on C cannot be greater than work done (by torque) on B ✓

$W = T\theta$, if θ is 4x greater, T cannot be more than 1/4✓

Or $T_C \times 4\theta_B = T_B\theta_B$ so $T_C = T_B/4$

OR

Force same on both/force cannot increase/ r_C is ¼ r_B ✓

$F \times r_C = F \times r_B/4$ so $TC = T_B/4$

Or Because radius is ¼, torque on C must be ¼ ✓

Accept other valid argument e.g. using knowledge that radius of C is 1/4 radius of B, or velocity v at point of mesh of gears is the same for both.

Do not allow 'it is not possible' (WTTE) unless backed up by valid argument.

2

(c) α = 76/2.1 = 36 (rad s^{-1}) (36.2 rad s^{-1}) ✓

$I = T/\alpha = \frac{0.054}{36}$ = 1.5 × 10^{-3} (kg m^2) ✓

ECF for 2nd mark for AE or transposing error.

2

(d) angular impulse = ang. momentum change = $T\Delta t$ ✓

1st mark for statement defining angular impulse

Reference to (large) $\Delta(I\omega)$ in small Δt gives large T ✓

2nd for relating momentum change in small t to high T

($T = F \times r$) so large F on gear teeth. ✓

3rd for relating high T to high force

3

[8]

10. (a) To smooth out (fluctuations in) rotational speed ✓

OR to store (rotational kinetic) energy ✓

OR to smooth (fluctuations in) torque/power ✓

Any named form of energy must be (rotational) kinetic

Do not allow an application (eg regenerative braking) unless one of the answers shown alongside is included

1

(b) Use of 0.075 sin 8° **or** 0.075 tan 8° **or** 0.075 (π − 3) to calculate h

h = 1.04 × 10^{-2} (m) **OR** 1.05 × 10^{-2} (m) **OR** 1.06 × 10^{-2} (m) ✓

mgh = 0.020 × 9.81 × 1.04 × 10^{-2} = 2.04 × 10^{-3} (J) ✓

$T\theta$ = 2.04 × 10^{-3} (J)

T = 2.04 × 10^{-3} /3.00 = 6.80 × 10^{-4} Nm ✓

1st mark for calculating h

2nd mark for calculating mgh.

3rd mark for dividing mgh by 3.00 rad.

Use of tan gives h = 1.05 × 10^{-2} (m)

Use of arc length gives h = 1.06 × 10^{-2} (m)

3rd mark only awarded for arriving at correct answer to more than 1 sig fig

3

(c) Attempt to use $0 = \omega_1^2 - 2\alpha\theta$ ✓
Or $\theta = 573 \times 2\pi = 3600$ rad ✓
Leading to $\alpha = 0.087$ (rad s^{-2}) ✓

$I = \frac{T}{\alpha} = 7.82 \times 10^{-3}$ kg m^2 ✓

OR
Attempt to use $T\theta = ½ I (\omega_2^2 - \omega_1^2)$ ✓
or $\theta = 573 \times 2\pi = 3600$ rad ✓

($I = 2T\theta/\omega_1^2$)
$= 2 \times 6.8 \times 10^{-4} \times 573 \times 2\pi/25^2$ ✓
$= 7.82 \times 10^{-3}$ kg m^2 ✓

*1st mark for **either** use of equation **or** converting rotations to rad*

ECF for 3rd mark

The value of torque used must be a correctly calculated answer to part (b) or 7×10^{-4} N m

<u>For 2nd method</u>

2nd mark for correct substitution

3rd mark for calculating answer

3

[7]