M1.(a) (i) Distance travelled in muons' frame of reference

= $10700(1-0.996^2)^{1/2}$ =956 m ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Time in Earth frame of reference

= 10700 / (0.996 × 3 × 10^8) = 3.581 × 10^{-5} s ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Half-life in Earth frame of reference

=1.6 × 10^{-6} / $(1-0.996^2)^{1/2}$ = 17.9× 10^{-6} s ✓

Time taken = 35.8 × 10^{-6} s ✓

This is 2 half lives so number reaching Earth = 250 ✓

OR

Distance travelled in muons' frame of reference

= $10700(1-0.996^2)^{1/2}$ =956 m ✓

Distance the muon travels in one half-life in muons reference frame

= 0.996 × 3 × 10^8 × 1.6 × 10^{-6} = 478 m ✓

Therefore 2 half-lives elapse to travel 956 m so number = 250 ✓

OR

Decay constant in muon frame of reference

Or decay constant in the Earth frame of reference ✓

Uses the corresponding elapsed time and decay constant in

$N = N_0 e^{-\lambda t}$ ✓

Arrives at 250 ✓

All steps in the working must be seen

Award marks according to which route they appear to be taking

The number left must be deduced from the correct time that has elapsed in the frame of reference they are using

3

(ii)

	✓ if correct
For an observer in a laboratory on Earth the distance travelled by a muon is greater than the distance travelled by the muon in its frame of reference	✓
For an observer in a laboratory on Earth time passes more slowly than for a muon in its frame of	

reference	
For an observer in a laboratory on Earth, the probability of a muon decaying each second is lower than it is for a muon in its frame of reference	

1

(b) (i) Total energy = $9.11 \times 10^{-31} \times (3 \times 10^{8})^{2} / (1\text{-}0.98^{2})^{1/2}$ ✓

4.12×10^{-13} J seen to 2 or more sf ✓

Show that so working must be seen

2

(ii) Change = 7.5×10^{-14} J

V = 469 (470) kV allow ecf using their answer to (i) ✓

ecf is their ((i) −3.37) × 10^{-13}) / 1.6 × 10^{-19}

Using 4 × 10^{-13} gives 394 (390) kV

Using 3.9 × 10^{-13} gives 331(330) kV

Do not allow 1 sf answer

1

[7]

M2.(a)

(Using $m = \dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$ gives)

$$2 = \frac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} \quad \text{or} \quad \sqrt{1-\frac{v^2}{c^2}} = 0.5 \ \checkmark$$

(Rearranging gives)

$v\ (= \sqrt{1-0.5^2}\ c) = 0.866\ c$ or 2.6×10^{8} m s^{-1} ✓

Accept either answer.

2

(b) curve starts at v=0, $m = m_o$ and rises smoothly ✓

2nd mark; ecf from a if plotted correctly

curve passes through $2m_o$ at v = 0.87 c (± 0.03*c* or in 2nd half of x-scale div containing 0.87c) ✓

3rd mark; There must be visible white space between the curve and the v = c line; also, the curve must reach $7m_o$ at least.

curve is asymptotic at $v = c$ (and does not cross or touch $v = c$ or curve back) ✓

3

(c) Energy = mc^2 so (as $v \to c$) energy of particle increases as mass increases ✓

Alternative scheme for 1 mark only; mass infinite at v = c which is (physically) <u>impossible</u> ✓

mass -> infinity as $v \to c$ so energy -> infinity which is (physically) <u>impossible</u> ✓

[OR for one mark only

force = *ma* so force increases as mass increases

Mass -> infinity as v->c so force -> infinity which is (physically) <u>impossible</u> ✓]

2

[7]

M3. (i) E_k (= eV) (= $1.6 \times 10^{-19} \times 1.1 \times 10^9$)
= 1.8×10^{-10} (J) **(1)** (1.76×10^{-10} (J))

(ii) (use of $E = mc^2$ gives) $\Delta m = \left(\dfrac{1.8 \times 10^{-10}}{(3 \times 10^8)^2}\right) = 2.0 \times 10^{-27}$ (kg) **(1)**

$= \dfrac{2.0 \times 10^{-27}}{1.67 \times 10^{-27}} m_0 = 1.2 m_0$ **(1)**

(allow C.E. for value of E_k from (i), but not 3rd mark)

$\therefore\ m = m_o + \Delta m$ **(1)** (= 2.2 m_o)

(iii) (use of $m = m_o \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$ gives) $2.2m_o = \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2}$ **(1)**

$v = \left(1 - \frac{1}{2.2^2}\right)^{1/2} c$ **(1)**

$= 2.7 \times 10^8$ m s^{-1} **(1)**

[7]

M4.(a) as speed $\rightarrow c$, mass $\rightarrow$ infinite **(1)**
gain of E_k causes large gain of mass when speed is close to c **(1)**
gain of E_k causes small gain of speed when speed is close to c **(1)**
$E_k = \frac{1}{2}mv^2$ valid at speeds << c **(1)**

max 3

The Quality of Written Communication marks are awarded for the quality of answers to this question.

(b) (i) $E_k = eV = 1.6 \times 10^{-19} \times 2.1 \times 10^{10}$ **(1)** (= $3.3(6) \times 10^{-9}$J)

(ii) (use of $m = \frac{E_k}{c^2}$ gives) gain of mass = $\frac{3.36 \times 10^{-9}}{(3 \times 10^8)^2}$ = 3.7×10^{-26}(kg) **(1)**

$= \frac{3.37 \times 10^{-26}}{1.67 \times 10^{-27}} m_0 = 22\ m_0$ **(1)**

mass of proton = $22\ m_0 + m_0$ **(1)** (=$23\ m_0$)

(using $E_k = 3.4 \times 10^{-9}$ gives gain of mass = 3.8×10^{-26} (kg) ≡ $23\ m_0$
mass of proton = $24\ m_0$

4

(c) $23 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$ **(1)**

$\frac{v^2}{c_2} = \left(1 - \frac{1}{25}\right)$ = 0.998 **(1)**

$v = 0.999\ c = 2.99(7) \times 10^8$ m s^{-1}

3

[10]

M5.(a) (i) $l = (vt = 1.00 \times 10^{8} \times 15 \times 10^{-9}) = 1.50\text{m}$ **(1)**

(ii) $\left(l = l_0\sqrt{1-\dfrac{v^2}{c^2}}\right)$

$1.50 = l_0 \sqrt{1-\dfrac{(1.00\times10^{8})^2}{(3.00\times10^{8})^2}}$ **(1)**

$l_0 \left(= \dfrac{1.50}{0.943}\right) = 1.59\text{ m}$ **(1)**

3

(b) (i) $m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$ **(1)** $\left[\text{or } \dfrac{m_0}{\sqrt{1-\dfrac{(1.00\times10^{8})^2}{(3.00\times10^{8})^2}}}\right]$

$m \left(\text{or } \dfrac{m_0}{\sqrt{1-\dfrac{(1.00\times10^{8})^2}{(3.00\times10^{8})^2}}}\right) = 1.06m_0$

[or = $1.06 \times 1.67 \times 10^{-27}$ or 1.77×10^{-27} kg] **(1)**
kinetic energy = $(m - m_0)c^2$ **(1)**
[or = $0.06m_0c^2$ or $0.06 \times 1.67 \times 10^{-27} \times (3 \times 10^{8})^2$]
= 9.1×10^{-12} (J) **(1)**

(ii) total k.e. = $(10^{7} \times 9.1 \times 10^{-12}) = 9.1 \times 10^{-5}$ (J) **(1)**
k.e. per second $\left(= \dfrac{9.1\times10^{-5}}{1.5\times10^{-9}}\right) = 6080\text{W}$

max 5
[8]