## AS Pure Mathematics 8MAO: Specimen Paper 1 Mark Scheme

| Question   | Scheme   | Marks    | AOs          |
|--|--|----------|--------------|
| 1 (a)  | $y = 2x^3 - 2x^2 - 2x + 8 \Longrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 4x - 2$   | M1<br>A1 | 1.1b<br>1.1b |
|  |  | (2)      |              |
| (b)  | Attempts $6x^2 - 4x - 2 > 0 \implies (6x + 2)(x - 1) > 0$  | M1       | 1.1b         |
|  | $x = -\frac{1}{3}, 1$  | A1       | 1.1b         |
|  | Chooses outside region   | M1       | 1.1b         |
|  | $\left\{x:x<-\frac{1}{3}\right\}\cup\left\{x:x>1\right\}$  | A1       | 2.5          |
|  |  | (4)      |              |
|  |  | (6 n     | narks)       |
| Notes:   |  |          |              |
| (a)<br>M1: Attemp                                | ots to differentiate. Allow for two correct terms un-simplified  |          |              |
| A1: $\frac{dy}{dx} =$                            | $=6x^2-4x-2$   |          |              |
| (b)  |  |          |              |
| M1: Attemp                                       | ots to find the critical values of their $\frac{dy}{dx} > 0$ or their $\frac{dy}{dx} = 0$  |          |              |
| A1: Correct                                      | critical values $x = -\frac{1}{3}, 1$  |          |              |
|  | es the outside region  |          |              |
| <b>A1:</b> $\begin{cases} x : x < x \end{cases}$ | $\left\{x = \frac{1}{3}\right\} \cup \left\{x : x > 1\right\} \text{ or } \left\{x : x \in \mathbb{R} \mid x < -\frac{1}{3} \text{ or } x > 1\right\}$ |          |              |
| Accept   | also $\left\{x:x,,-\frac{1}{3}\right\}\cup\left\{x:x1\right\}$   |          |              |

| Question                         | Scheme  | Marks              | AOs    |
|----------------------------------|---|--------------------|--------|
| 2 (a)                            | $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\mathbf{i} - 3\mathbf{j} - (4\mathbf{i} + 2\mathbf{j})$   | M1                 | 1.1b   |
|                                  | =2i-5j  | A1                 | 1.1b   |
|                                  |   | (2)                |        |
| 1(b)                             | Explains that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ as $8\mathbf{i} - 20\mathbf{j} = 4 \times (2\mathbf{i} - 5\mathbf{j})$   | M1                 | 1.1b   |
|                                  | As $\overrightarrow{OC} = 4 \times \overrightarrow{AB}$ it is parallel to it and not the same length<br>Hence $OABC$ is a trapezium.  | A1                 | 2.4    |
|                                  |   | (2)                |        |
|                                  |   | (4 n               | narks) |
| Notes:                           |   |                    |        |
| (a)<br>M1: Attemp<br>A1: 2i – 5j | ots $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or equivalent. This may be implied by one correct co  | mponent            |        |
| (b)                              | $\longrightarrow$ $\longrightarrow$   |                    |        |
| A1: Fully ex                     | ots to compare vectors $\overrightarrow{OC}$ and $\overrightarrow{AB}$ by considering their directions splains why $OABC$ is a trapezium. (The candidate is required to state the same length as it.) | at <i>OC</i> is pa | rallel |

| Question  | Scheme  | Marks | AOs    |
|---|---|-------|--------|
| <b>3</b> (a)  | Uses or implies that $V = at + b$   | B1    | 3.3    |
|   | Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get either a or b  | M1    | 3.1b   |
|   | Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get both a and b   | M1    | 1.1b   |
|   | $\Rightarrow V = -\frac{1}{30}t + 4.8$  | A1    | 1.1b   |
|   |   | (4)   |        |
| <b>(b)</b>  | (i) States that the initial volume is 4.8 m <sup>3</sup>  | B1 ft | 3.4    |
|   | (ii) Attempts to solve $0 = -\frac{1}{30}t + 4.8$   | M1    | 3.4    |
|   | States 144 minutes  | A1    | 1.1b   |
|   |   | (3)   |        |
| (c)   | <ul> <li>States any logical reason</li> <li>The tank will leak more quickly at the start due to the greater water pressure</li> <li>The hole will probably get larger and so will start to leak more quickly</li> <li>Sediment could cause the leak to be plugged and so the tank would not empty.</li> </ul>   | B1    | 3.5b   |
|   |   | (1)   |        |
|   | ·   | (8 n  | narks) |
| Notes:  |   |       |        |
| You may av<br>M1: Award<br>4 = 24a + b<br>may just see<br>M1: Uses 4<br>A1: $V = -\frac{2}{3}$<br>(b)(i)<br>B1ft: Follow<br>(b)(ii) | The implies that $V = at + b$<br>word this at their final line but it must be $V = f(t)$<br>ded for translating the problem in context and starting to solve. It is score<br>and $2.8 = 60a + b$ are written down and the candidate proceeds to find e<br>e a line $\pm \frac{4-2.8}{60-24}$<br>k = 24a + b and $2.8 = 60a + b$ to find both a and b<br>$\frac{1}{50}t + 4.8$ or exact equivalent. Eg $30V + t = 144$<br>w through on their 'b'<br>that $V = 0$ and finds t by attempting to solve their $0 = -\frac{1}{30}t + 4.8$ |       |        |
| A1: States 1  |   |       |        |
| (c)   |   |       |        |

| Question   | Scheme               | Marks AO |
|------------|----------------------|----------|
| 4(a)       | (4,-3)               | B1 1.2   |
|            |                      | (1)      |
| (b)        | x = 6                | B1 1.11  |
|            |                      | (1)      |
| (c)        | <i>x</i> ,, <b>4</b> | B1 1.11  |
|            |                      | (1)      |
| (d)        | k >1.5               | B1 2.2a  |
|            |                      | (1)      |
|            | ·                    | (4 marks |
| Notes:     |                      |          |
| See m/sche | me                   |          |
|            |                      |          |
|            |                      |          |

| Question          | Scheme   | Marks      | AOs    |
|-------------------|--|------------|--------|
| 5(a)              | $f(-3) = (-3)^3 + 3 \times (-3)^2 - 4 \times (-3) - 12$  | M1         | 1.1b   |
|                   | $f(-3) = 0 \Rightarrow (x+3)$ is a factor $\Rightarrow$ Hence $f(x)$ is divisible by $(x+3)$ .   | A1         | 2.4    |
|                   |  | (2)        |        |
| (b)               | $x^{3} + 3x^{2} - 4x - 12 = (x+3)(x^{2} - 4)$  | M1         | 1.1b   |
|                   | =(x+3)(x+2)(x-2)   | dM1        | 1.1b   |
|                   |  | A1         | 1.1b   |
|                   |  | (3)        |        |
| (c)               | $\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = \frac{\dots}{x(x^2 + 5x + 6)}$   | M1         | 3.1a   |
|                   | $=\frac{(x+3)(x+2)(x-2)}{x(x+3)(x+2)}$   | dM1        | 1.1b   |
|                   | $=\frac{(x-2)}{x}=1-\frac{2}{x}$   | A1         | 2.1    |
|                   |  | (3)        |        |
|                   |  | (8 n       | narks) |
| Notes:            |  |            |        |
| (b)<br>M1: Attemp | tots $f(-3)$<br>we $f(-3) = 0$ and explains that $(x+3)$ is a factor and hence $f(x)$ is division<br>but to divide by $(x+3)$ to get the quadratic factor.<br>ision look for the first two terms. ie $x^2 + 0x$<br>$x+3)\frac{x^2 \pm 0x}{x^3 + 3x^2 - 4x - 12}$ | ble by (x- | +3).   |
| By insp           | ection look for the first and last term $x^3 + 3x^2 - 4x - 12 = (x+3)(x^2 +x)$   | ±4)        |        |
|                   | n attempt at factorising their $(x^2 - 4)$ . (Need to check first and last terms   |            |        |
|                   | (x+3)(x+2)(x-2)  | ,          |        |
| (c)<br>M1: Takes  | a common factor of x out of the denominator and writes the numerator in<br>atively rewrites to $x^3 + 3x^2 - 4x - 12 = A(x^3 + 5x^2 + 6x) + B(x^2 + 5x + 6)$   |            |        |
|                   | er factorises the denominator and cancels  | ,          |        |
|                   | natively compares terms or otherwise to find either A or $B$   |            |        |
| A1: Shows         | that $\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = 1 - \frac{2}{x}$ with no errors or omissions  |            |        |

In the alternative there must be a reference to

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$$x^{3} + 3x^{2} - 4x - 12 \equiv 1 \left( x^{3} + 5x^{2} + 6x \right) - 2 \left( x^{2} + 5x + 6 \right) \text{ and hence } \frac{x^{3} + 3x^{2} - 4x - 12}{x^{3} + 5x^{2} + 6x} = 1 - \frac{2}{x}$$

| Question | Scheme   | Marks | AOs   |
|----------|--|-------|-------|
| 6(i)     | Tries at least one value in the interval<br>Eg $4^2 - 4 - 1 = 11$  | M1    | 1.18  |
|          | States that when $n = 8$ it is FALSE and provides evidence<br>$8^2 - 8 - 1 = 55 = (11 \times 5)$ Hence NOT PRIME | A1    | 2.4   |
|          |  | (2)   |       |
| (ii)     | Knows that an odd number is of the form $2n+1$   | B1    | 3.1   |
|          | Attempts to simplify $(2n+1)^3 - (2n+1)^2$   | M1    | 2.1   |
|          | and factorise $8n^3 + 8n^2 + 2n = 2(4n^3 + 4n^2 + 1n) =$   | dM1   | 1.1   |
|          | with statement $2 \times$ is always even   | A1    | 2.4   |
|          |  | (4)   |       |
| Alt (ii) | Let the odd number be 'n' and attempts $n^3 - n^2$   | B1    | 3.1   |
|          | Attempts to factorise $n^3 - n^2 = n^2 (n-1)$  | M1    | 2.1   |
|          | States that $n^2$ is odd (odd × odd) and $(n-1)$ is even (odd -1)  | dM1   | 1.1   |
|          | States that the product is even ( odd×even)  | A1    | 2.4   |
|          |  | (6 n  | narks |

## (i)

M1: Attempts any  $n^2 - n - 1$  for *n* in the interval. It is acceptable just to show  $8^2 - 8 - 1 = 55$ A1: States that when n = 8 it is FALSE and provides evidence. A comment that  $55 = 11 \times 5$  and hence not prime is required

## (ii)

## See scheme for two examples of proof

Note that Alt (i) works equally well with an odd number of the form 2n-1

| Question                                  | Scheme   | Marks              | AOs          |
|---|--|--------------------|--------------|
| 7 (a)                                     | $\left(1+\frac{3}{x}\right)^2 = 1+\frac{6}{x}+\frac{9}{x^2}$   | M1<br>A1           | 1.1b<br>1.1b |
|   |  | (2)                |              |
| (b)                                       | $\left(1+\frac{3}{4}x\right)^{6} = 1+6 \times \left(\frac{3}{4}x\right) + \dots$   | B1                 | 1.1b         |
|   | $\left(1 + \frac{3}{4}x\right)^{6} = 1 + 6 \times \left(\frac{3}{4}x\right) + \frac{6 \times 5}{2} \times \left(\frac{3}{4}x\right)^{2} + \frac{6 \times 5 \times 4}{3 \times 2} \times \left(\frac{3}{4}x\right)^{3} + \dots$ | M1<br>A1           | 1.1b<br>1.1b |
|   | $=1+\frac{9}{2}x+\frac{135}{16}x^2+\frac{135}{16}x^3+\dots$  | A1                 | 1.1b         |
|   |  | (4)                |              |
| (c)                                       | $\left(1+\frac{3}{x}\right)^2 \left(1+\frac{3}{4}x\right)^6 = \left(1+\frac{6}{x}+\frac{9}{x^2}\right) \left(1+\frac{9}{2}x+\frac{135}{16}x^2+\frac{135}{16}x^3+\dots\right)$  |                    |              |
|   | Coefficient of $x = \frac{9}{2} + 6 \times \frac{135}{16} + 9 \times \frac{135}{16} = \frac{2097}{16}$   | M1<br>A1           | 2.1<br>1.1b  |
|   |  | (2)                |              |
|   |  | (8 n               | narks)       |
| Notes:                                    |  |                    |              |
|   | $\operatorname{pts}\left(1+\frac{3}{x}\right)^2 = A + \frac{B}{x} + \frac{C}{x^2}$ $\frac{3}{x}\right)^2 = 1 + \frac{6}{x} + \frac{9}{x^2}$  |                    |              |
| M1: Attemp<br>least once in<br>A1: Binomi | o terms correct, may be un-simplified<br>ots the binomial expansion. Implied by the correct coefficient and power<br>a term 3 or 4<br>al expansion correct and un-simplified<br>al expansion correct and simplified.           | of <i>x</i> seen a | at           |
| (c)                                       | nes all relevant terms for their $\left(1 + \frac{A}{x} + \frac{B}{x^2}\right)\left(1 + Cx + Dx^2 + Ex^3 +\right)$ to find   | nd the             |              |
| coefficient of <b>A1:</b> Fully co        | of x.  |                    |              |

7

PMT

| Question   | Scheme  | Marks      | AOs          |
|--|---|------------|--------------|
| 8(a)   | (i) $\int_{1}^{a} \sqrt{8x}  dx = \sqrt{8} \times \int_{1}^{a} \sqrt{x}  dx = 10\sqrt{8} = 20\sqrt{2}$  | M1<br>A1   | 2.2a<br>1.1b |
|  | (ii) $\int_{0}^{a} \sqrt{x}  dx = \int_{0}^{1} \sqrt{x}  dx + \int_{1}^{a} \sqrt{x}  dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_{0}^{1} + 10 = \frac{32}{3}$  | M1<br>A1   | 2.1<br>1.1b  |
|  |   | (4)        |              |
| (b)  | $R = \int_{1}^{a} \sqrt{x}  \mathrm{d}x = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_{1}^{a}$  | M1<br>A1   | 1.1b<br>1.1b |
|  | $\frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3} = 10 \Longrightarrow a^{\frac{3}{2}} = 16 \Longrightarrow a = 16^{\frac{2}{3}}$   | dM1        | 3.1a         |
|  | $\Rightarrow a = 2^{4 \times \frac{2}{3}} = 2^{\frac{8}{3}}$  | Al         | 2.1          |
|  |   | (4)        |              |
| Notes:   |   | (8 n       | narks)       |
| (a)(ii)<br>M1: For ide<br>A1: For $\frac{32}{3}$ | entifying and attempting to use $\int_{0}^{a} \sqrt{x}  dx = \int_{0}^{1} \sqrt{x}  dx + \int_{1}^{a} \sqrt{x}  dx$ or exact equivalent   |            |              |
| <b>A1:</b> $\int_1^a $                           | but to integrate, $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$<br>$\overline{x}  dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_{1}^{a}$   |            |              |
| limits and p                                     | whole strategy to find <i>a</i> . In the scheme it is awarded for setting $\left[ \dots x^{\frac{3}{2}} \right]_{1}^{a}$ roceeding using correct index work to find <i>a</i> . Alternatively a candidate c uch a case it is awarded for setting $\left[ \dots x^{\frac{3}{2}} \right]^{2^{k}} = 10$ , using both limits and p | ould assun | ne           |
|  | x work to $k=$  |            |              |
|  | ative case, a further statement must be seen following $k = \frac{8}{3}$ Hence Tru  | e          |              |

8

PMT

| Question       | Scheme   | Marks  | AOs    |  |  |
|----------------|--|--|--------|--|--|
| 9              | $2\log_4(2-x) - \log_4(x+5) = 1$   |  |        |  |  |
|                | Uses the power law $\log_4 (2-x)^2 - \log_4 (x+5) = 1$   | M1   | 1.1b   |  |  |
|                | Uses the subtraction law $\log_4 \frac{(2-x)^2}{(x+5)} = 1$  | M1   | 1.1b   |  |  |
|                | $\frac{(2-x)^2}{(x+5)} = 4 \rightarrow 3\text{TQ in } x$   | dM1  | 3.1a   |  |  |
|                | $x^2 - 8x - 16 = 0$  | A1   | 1.1b   |  |  |
|                | $(x-4)^2 = 32 \Longrightarrow x =$   | M1   | 1.1b   |  |  |
|                | $x = 4 - 4\sqrt{2}$ oe only  | A1   | 2.3    |  |  |
|                |  | (6)  |        |  |  |
|                |  | (6 n   | narks) |  |  |
| Notes:         |  |  |        |  |  |
| M1: Uses th    | he power law of logs $2\log_4(2-x) = \log_4(2-x)^2$  |  |        |  |  |
| M1: Uses th    | he subtraction law of logs following the above $\log_4(2-x)^2 - \log_4(x+5) = \log_4(x+5)$   | $\mathbf{g}_4 \frac{\left(2-x\right)^2}{\left(x+5\right)}$ |        |  |  |
| Alternativel   | Alternatively uses the addition law following use of $1 = \log_4 4$ That is $1 + \log_4 (x+5) = \log_4 4(x+5)$   |  |        |  |  |
| correct use of | <b>dM1:</b> This can be awarded for the overall strategy leading to a $3TQ$ in $x$ . It is dependent upon the correct use of both previous M's and for undoing the logs to reach a $3TQ$ equation in $x$ |  |        |  |  |
|                | A1: For a correct equation in $x$<br>M1: For the correct method of solving their $3TQ = 0$   |  |        |  |  |
|                | $-4\sqrt{2}$ or exact equivalent only. (For example accept $x = 4 - \sqrt{32}$ )   |  |        |  |  |

| Question  | Scheme  | Marks         | AOs    |
|---|---|---------------|--------|
| 10(a)   | Attempts to find the radius $\sqrt{(2-2)^2+(5-3)^2}$ or radius <sup>2</sup>   | M1            | 1.1b   |
|   | Attempts $(x-2)^{2} + (y-5)^{2} = 'r^{2}$   | M1            | 1.1b   |
|   | Correct equation $(x-2)^{2} + (y-5)^{2} = 20$   | A1            | 1.1b   |
|   |   | (3)           |        |
| (b)   | Gradient of radius <i>OP</i> where <i>O</i> is the centre of $C = \frac{5-3}{2-2} = \left(\frac{1}{2}\right)$   | M1            | 1.1b   |
|   | Equation of <i>l</i> is $-2 = \frac{y-3}{x+2}$  | dM1           | 3.1a   |
|   | Any correct form $y = -2x - 1$  | A1            | 1.1b   |
|   | Method of finding k Substitute $x=2$ into their $y=-2x-1$   | M1            | 2.1    |
|   | k = -5  | A1            | 1.1b   |
|   |   | (5)           |        |
|   |   | (8 n          | narks) |
| Notes:  |   |               |        |
| A1: For a co<br>If students u<br>A1: $x^2 + y^2$<br>(b)<br>M1: Attempt          | eme or substitutes $(-2,3)$ into $(x-2)^2 + (y-5)^2 = r^{2}$<br>prect equation<br>use $x^2 + y^2 + 2fx + 2gy + c = 0$ M1: $f = 2, g = 5$ M1: substitutes (2,5) the $x^2 - 4x - 10y + 9 = 0$<br>puts to find the gradient of <i>OP</i> where <i>O</i> is the centre of <i>C</i><br>a complete strategy of finding the equation of <i>l</i> using the perpendicular generative (-2,3) |               |        |
|   | rrect form of $l$ Eg $y = -2x - 1$  |               |        |
| their $y = -2$<br>A1: $k = -5$<br>Alt using P<br>M1: Attemp<br>dM1: For their k | for the key step of finding k. In this method they are required to substitut $x-1$ and solve for k.<br><b>ythagoras' theorem</b><br>ots Pythagoras to find both PQ and OQ in terms of k (where O is centre of the complete strategy of using Pythagoras theorem on triangle POQ to form   | of <i>C</i> ) |        |
|   | for a correct attempt to solve their quadratic to find k.   |               |        |
|   |   |               |        |

| Question   | Scheme   | Marks | AOs    |
|--|--|-------|--------|
| 11(i)  | $(2\theta + 10^\circ) = \arcsin(-0.6)$   | M1    | 1.1b   |
|  | $(2\theta+10^\circ) = -143.13^\circ, -36.87^\circ, 216.87^\circ, 323.13^\circ$ (Any two)   | A1    | 1.1b   |
|  | Correct order to find $\theta = \dots$   | dM1   | 1.1b   |
|  | Two of $\theta = -76.6^{\circ}, -23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}.$  | A1    | 1.1b   |
|  | $\theta = -76.6^{\circ}, -23.4^{\circ}, 103.4^{\circ}, 156.6^{\circ}, \text{ only}$  | A1    | 2.1    |
|  |  | (5)   |        |
| (ii)   | (a) Explains that the student has not considered the negative value of $x(=-29.0^{\circ})$ when solving $\cos x = \frac{7}{8}$   | B1    | 2.3    |
|  | Explains that the student should check if any solutions of $\sin x = 0$ (the cancelled term) are solutions of the given equation. $x = 0^{\circ}$ should have been included as a solution  | B1    | 2.3    |
|  | (b) Attempts to solve $4\alpha + 199^{\circ} = (360 - 29.0)^{\circ}$   | M1    | 2.2a   |
|  | $\alpha = 33.0^{\circ}$  | A1    | 1.1b   |
|  |  | (4)   |        |
|  |  | (9 n  | narks) |
| Notes:   |  |       |        |
| A1: Any tw<br>dM1: Corre<br>A1: Any tw<br>A1: A full s | ots $\arcsin(-0.6)$ implied by any correct answer<br>to of $-143.13^\circ$ , $-36.87^\circ$ , $216.87^\circ$ , $323.13^\circ$<br>ect method to find any value of $\theta$<br>to of $\theta = -76.6^\circ$ , $-23.4^\circ$ , $103.4^\circ$ , $156.6^\circ$ .<br>Solution leading to all four answers and no extras<br>$.6^\circ$ , $-23.4^\circ$ , $103.4^\circ$ , $156.6^\circ$ , only |       |        |
| (ii)(a)<br>B1: See sch<br>B1: See sch                  | eme  |       |        |
| (ii)(b)<br><b>M1:</b> For de                           | ducing the smallest positive solution occurs when $4\alpha + 199^\circ = (360 - 29)^\circ$   | 0.0)° |        |
| A1: $\alpha = 33^{\circ}$                              | D  |       |        |

| Question | Scheme   |   | Marks | AOs  |
|----------|--|---|-------|------|
| 12(a)    | Sets $3x - 2\sqrt{x} = 8x - 16$                              |   | B1    | 1.1a |
|          | $2\sqrt{x} = 16 - 5x$ $4x = (16 - 5x)^2 \Longrightarrow x =$ | $5x + 2\sqrt{x} - 16 = 0$ $\Rightarrow (5\sqrt{x} \pm 8)(\sqrt{x} \pm 2) = 0$ | M1    | 3.1a |
|          | $25x^2 - 164x + 256 = 0$                                     | $\left(5\sqrt{x}-8\right)\left(\sqrt{x}+2\right)=0$                           | A1    | 1.1t |
|          | $(25x-64)(x-4) = 0 \Longrightarrow x =$                      | $\sqrt{x} = \frac{8}{5}, (-2) \Longrightarrow x = \dots$                      | M1    | 1.1t |
|          | $x = \frac{64}{25}$  | only  | A1    | 2.3  |
|          |  |   | (5)   |      |
| (b)      | Attempts to solve $3x - 2\sqrt{x} = 0$                       |   | M1    | 2.1  |
|          | Correct solution $x = \frac{4}{9}$                           |   | A1    | 1.1t |
|          | $y_{,,,} 3x - 2\sqrt{x}, y > 8x - 16 x \dots \frac{4}{9}$    |   | B1ft  | 1.1t |
|          |  |   | (3)   |      |

Notes:

(a)

**B1:** Sets the equations equal to each other and achieves a correct equation

M1: Awarded for the key step in solving the problem. This can be awarded via two routes. Both routes must lead to a value for x.

- Making the  $\sqrt{x}$  term the subject and squaring both sides (not each term)
- Recognising that this is a quadratic in  $\sqrt{x}$  and attempting to factorise  $\Rightarrow (5\sqrt{x}\pm 8)(\sqrt{x}\pm 2)=0$

A1: A correct intermediate line  $25x^2 - 164x + 256 = 0$  or  $(5\sqrt{x} - 8)(\sqrt{x} + 2) = 0$ 

**M1:** A correct method to find at least one value for *x*. Way One it is for factorising (usual rules), Way Two it is squaring at least one result of their  $\sqrt{x}$ 

A1: Realises that  $x = \frac{64}{25}$  is the only solution  $x = \frac{64}{25}$ , 4 is A0 (b) M1: Attempts to solve  $3x - 2\sqrt{x} = 0$  For example Allow  $3x = 2\sqrt{x} \Rightarrow 9x^2 = 4x \Rightarrow x = ...$ Allow  $3x = 2\sqrt{x} \Rightarrow x^{\frac{1}{2}} = \frac{2}{3} \Rightarrow x = ...$ A1: Correct solution to  $3x - 2\sqrt{x} = 0 \Rightarrow x = \frac{4}{9}$ B1: For a consistent solution defining *R* using either convention Either *y*,  $3x - 2\sqrt{x}$ ,  $y > 8x - 16 x ... \frac{4}{9}$  Or  $y < 3x - 2\sqrt{x}$ ,  $y ... 8x - 16 x > \frac{4}{9}$ 

| Question                      | Scheme  | Marks    | AOs          |
|-------------------------------|---|----------|--------------|
| 13(a)                         | $0.2 \mathrm{m}^2$  | B1       | 3.4          |
|                               |   | (1)      |              |
| (b)                           | $A = 0.2e^{0.3t}$ Rate of change = gradient = $\frac{dA}{dt} = 0.06e^{0.3t}$  | M1       | 3.1b         |
|                               | At $t = 5 \implies$ Rate of Growth is $0.06e^{1.5} = 0.269 \text{ m}^2/\text{day}$  | A1       | 1.1b         |
|                               |   | (2)      |              |
| (c)                           | $100 = 0.2e^{0.3t} \Longrightarrow e^{0.3t} = 500$  | M1<br>A1 | 3.1a<br>1.1b |
|                               | $\Rightarrow t = \frac{\ln(500)}{0.3} = 20.7 \text{ days} \qquad 20 \text{ days } 17 \text{ hours}$   | M1<br>A1 | 1.1b<br>3.2a |
|                               |   | (4)      |              |
|                               | At $t = 5 \Rightarrow$ Rate of Growth is $0.06e^{1.5} = 0.269 \text{ m}^2/\text{day}$   | A1       | 1.1b         |
|                               |   | (2)      |              |
| (d)                           | The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only 90% covered by the end of one month (28/29/30/31 days). Hence the model is not accurate | B1       | 3.5a         |
|                               |   | (1)      |              |
|                               |   | (8 n     | narks)       |
| Notes:                        |   |          |              |
| (a)                           |   |          |              |
| <b>B1:</b> 0.2 m <sup>2</sup> | oe  |          |              |
| (b)                           |   |          |              |
|                               | rate of change to gradient and differentiates $0.2e^{0.3t} \rightarrow ke^{0.3t}$   |          |              |
| A1: Correct (c)               | answer 0.269 m <sup>2</sup> /day  |          |              |
|                               | sutes $A = 100$ and proceeds to $e^{0.3t} = k$  |          |              |
| <b>A1:</b> $e^{0.3t} =$       | -   |          |              |
|                               | t method when proceeding from $e^{0.3t} = k \Longrightarrow t =$  |          |              |
| A1: 20 days                   |   |          |              |
| (d)                           |   |          |              |
| B1: Valid c                   | onclusion following through on their answer to (c).   |          |              |
|                               |   |          |              |

| Question | Scheme   | Marks | AOs  |  |
|----------|--|-------|------|--|
| 14       | $y = (x-2)^{2} (x+3) = (x^{2}-4x+4)(x+3) = x^{3}-1x^{2}-8x+12$   | B1    | 1.1b |  |
|          | An attempt to find x coordinate of the maximum point. To score this<br>you must see either<br>• an attempt to expand $(x-2)^2(x+3)$ , an attempt to<br>differentiate the result, followed by an attempt at solving<br>$\frac{dy}{dx} = 0$<br>• an attempt to differentiate $(x-2)^2(x+3)$ by the product rule<br>followed by an attempt at solving $\frac{dy}{dx} = 0$ | M1    | 3.1a |  |
|          | $y = x^3 - 1x^2 - 8x + 12 \Longrightarrow \frac{dy}{dx} = 3x^2 - 2x - 8$   | M1    | 1.1b |  |
|          | Maximum point occurs when $\frac{dy}{dx} = 0 \Rightarrow (x-2)(3x+4) = 0$  | M1    | 1.1b |  |
|          | $\Rightarrow x = -\frac{4}{3}$   | A1    | 1.1b |  |
|          | An attempt to find the area under $y = (x-2)^2 (x+3)$ between two values. To score this you must see an attempt to expand $(x-2)^2 (x+3)$ followed by an attempt at using two limits   | M1    | 3.1a |  |
|          | Area = $\int (x^3 - 1x^2 - 8x + 12) dx = \left[\frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x\right]$  | M1    | 1.1b |  |
|          | Uses a top limit of 2 and a bottom limit of their<br>$x = -\frac{4}{3} R = \left[\frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x\right]_{-\frac{4}{3}}^2$   | M1    | 2.2a |  |
|          | $Uses = \frac{28}{3} - \frac{1744}{81} = \frac{2500}{81}$  | A1    | 2.1  |  |
|          |  | (9)   |      |  |
|          |  |       |      |  |

**B1:** Expands  $(x-2)^2(x+3)$  to  $x^3-1x^2-8x+12$  seen at some point in their solution. It may appear just on their integral for example.

M1: This is a problem solving mark for knowing the method of finding the maximum point. You should expect to see the key points used (i) differentiation (ii) solution of their  $\frac{dy}{dx} = 0$ 

M1: For correctly differentiating their cubic with at least two terms correct (for their cubic).

M1: For setting their  $\frac{dy}{dx} = 0$  and solves using a correct method (including calculator methods)

A1: 
$$\Rightarrow x = -\frac{4}{3}$$

M1: This is a problem solving mark for knowing how integration is used to find the area underneath a curve between two points.

M1: For correctly integrating their cubic with at least two correct terms (for their cubic).

M1: For deducing the top limit is 2, the bottom limit is their  $x = -\frac{4}{3}$  and applying these correctly within their integration.

A1: Shows above steps clearly and proceeds to  $R = \frac{2500}{81}$