Level 1 / Level 2 GCSE (9 – 1)

MATHEMATICS

Paper 1 (Non- calculator)

Higher Tier

Time : 1 hour 30 minutes **Paper : 1 MA1 / 1H**

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

Information

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1. There are only red discs, blue discs, white discs and black discs in a box.

The table shows the probability of taking at random a red disc from the box.

Colour	Red	Blue	White	Black
Probability	0.1	0.2	0.6	0.1

The number of black discs in the box is the same as the number of red discs in the box.

The number of white discs in the box is three times the number of blue discs in the box.

a. Complete the table

P(Red)=P(Black)

$White = 3Blue$ let $P(\text{Blue}) = x$ then $P(\text{White}) = 3x$

$0.1 + x + 3x + 0.1 = 1$ (1)

$4x = 0.8$

$x = 0.2$ (1)

(2)

There are 8 red discs in the box.

b. Work out the total number of discs in the box.

0.1=10%

10% of the discs are red and there are 8 red discs, so there are $8 \times 10 = 80$ in total.

OR

$\frac{1 \times 8}{0.1} = 80$

(1) (1)

……..

(2)

(Total for Question 1 is 4 marks)

2. A person on holiday in the North Macedonia changed 480∈ into Macedonian denar when the exchange rate was 1MKD = 0.016 Euros. His total expenses were 3500 MKD per night for the three nights. And his other expenses were 8000 MKD for meal, 2275 MKD for travel costs. On returning home he changed what denar he had left into euro at a rate of 61.5 MKD = 1 Euro

Calculate

a. the number of denars received for the 480∈.

$\times 30000$ ($1\ \text{MKD} = 0.016\ \text{Euro}$; $30000\ \text{MKD} = 480\ \text{Euro}$) $\times 30000$

$\frac{480 \times 1}{0.016} = 30000$

……

(2)

b. The total amount spent, in denars.

$$(3500 \times 3) + 8000 + 2275 = 20775$$

……..

(1)

c. The amount in euros obtained for the denars he had left.

$$30000 - 20775 = 9225$$

$\times 150$ ($61.5\ MKD = 1\ Euro$; $9225\ MKD = 150$) $\times 150$

$\frac{9225 \times 1}{61.5} = 150\ Euro$

(1) (1)

……

(2)

(Total for Question 2 is 5 marks)

3. Find the lowest common multiple (LCM) of 30 and 48.

$30 = 2 \times 3 \times 5$ $48 = 2^4 \times 3$ (1)

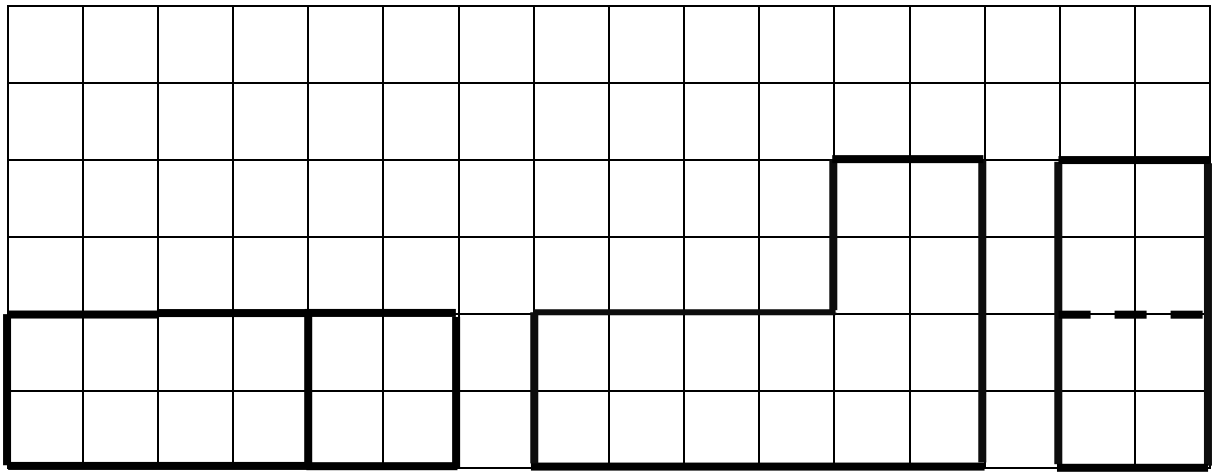
$\text{LCM} = 2^4 \times 3 \times 5 = 240$ (1)

……..

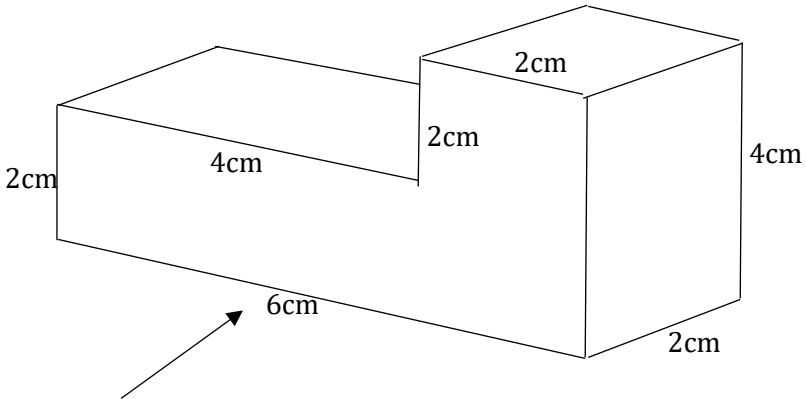
(2)

(Total for Question 3 is 2 marks)

4. Here is the plan, front elevation and side elevation views of a prism, drawn on cm squared paper.



In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



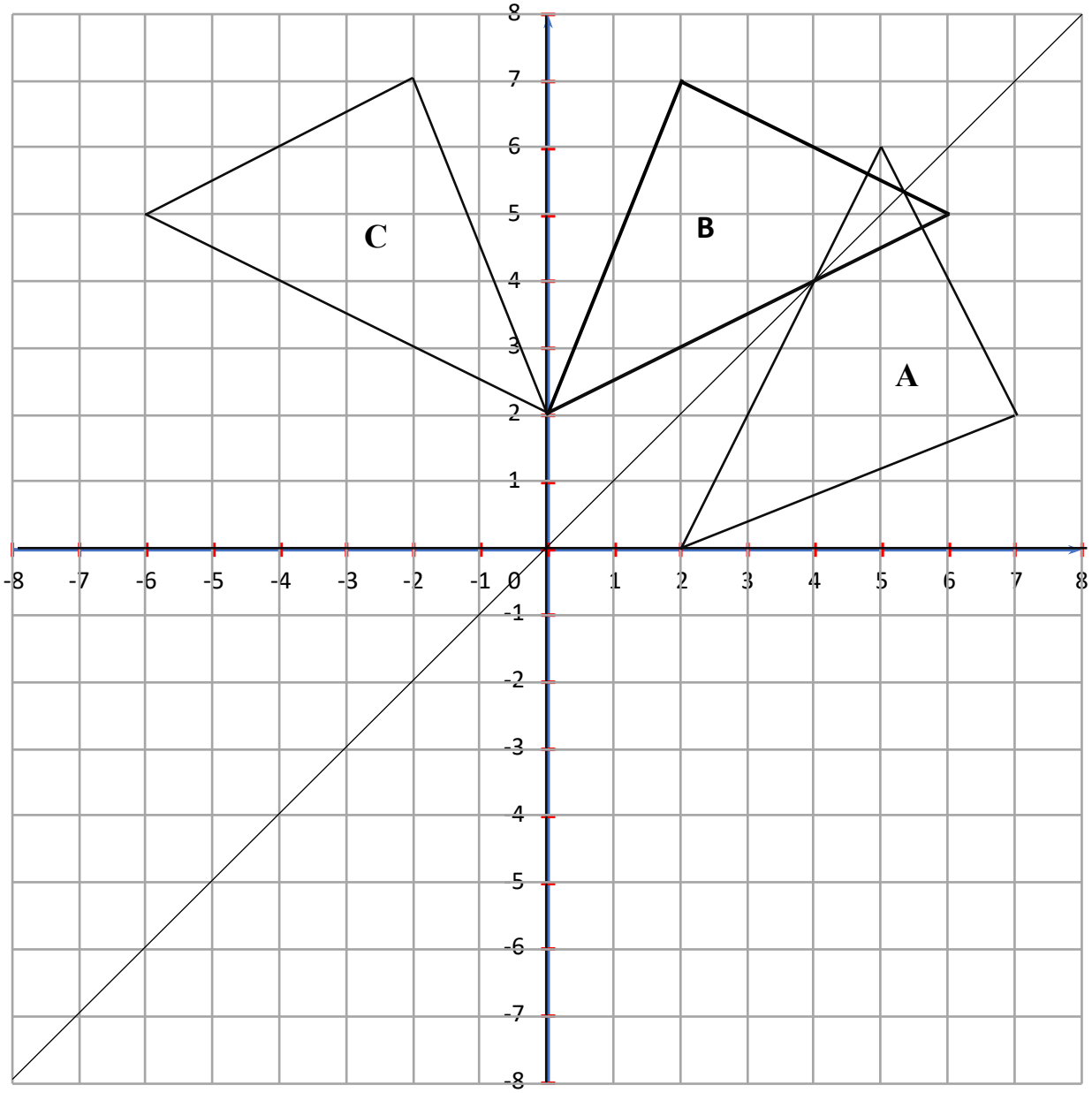
(1) Correct front shape

(1) Correct 3-D shape

(2)

(Total for Question 4 is 2 marks)

5.



a. Reflect triangle **A** in the line $y = x$. Label the image **B**.

(2)

b. Describe fully the single transformation that maps **C** onto **A**.

Rotation 90^0 anticlockwise about the origin.

(1) (1) (1)

(3)

(Total for Question 5 is 5 marks)

6. £1800 is raised and divided among 3 charities *P*, *Q*, *R* in the proportion 3 : 5 : 7.

a. Find the amount each charity receives.

The money is divided into $3 + 5 + 7 = 15$ parts

Value of one part: $\frac{1800}{15} = £120$ (1)

Amount P receives: $£120 \times 3 = £360$

Amount Q receives: $£120 \times 5 = £600$

Amount R receives: $£120 \times 7 = £840$

(1) any two correct values (1) all three correct values

……..

(3)

b. If these amounts are represented on a pie chart, calculate the angle of each sector.

Number of degrees for one part: $\frac{360^0}{15} = 24^0$ (1)

Angle of sector for P: $24 \times 3 = 72^0$

Angle of sector for Q: $24 \times 5 = 120^0$

Angle of sector for R: $24 \times 7 = 168^0$

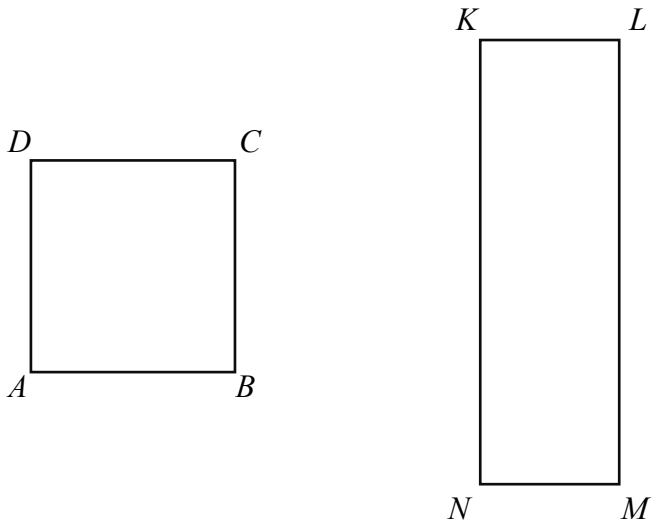
(1) any two correct values (1) all three correct values

……..^{0}

(3)

(Total for Question 6 is 6 marks)

7. Here is a diagram of a square and a rectangle.



$KL = 8$ cm

The perimeter of a square is 48 cm.

The area of *KLMN* is the same as the area of *ABCD*. Find the length of *LM*.

Let AB = a

$4a = 48$

$a = 12$ (1)

Area KLMN = Area ABCD

$8 \times LM = 12 \times 12$ (1)

$LM = \frac{144}{8}$ (1)

$LM = 18$ (1)

..... cm

(4)

(Total for Question 7 is 4 marks)

8. a. Work out an estimate of the value of $(13.2 \times (7.1 - 4.9))^2$

$(13 \times (7 - 5))^2$ (1) any 2 correct values of either 13, 7 or 5

$26^2 = 676$ (1)

……..

(2)

b. Find the value of $16^{-\frac{1}{2}}$

$\frac{1}{16^{\frac{1}{2}}} = \frac{1}{4}$ (1)

……..

(1)

c. The Nile is the longest river in Africa, which has a length of about 6758 km. Write the length correct to 1 significant figure.

7000 (1)

…… km

(1)

(Total for Question 1 is 4 marks)

9. Work out $3\frac{3}{8} \div 1\frac{1}{6}$

Give your answer as a mixed number in its simplest form.

$\frac{27}{8} \div \frac{7}{6}$ (1)

$\frac{27}{8} \times \frac{6}{7} = \frac{162}{56} = \frac{81}{28}$ (1)

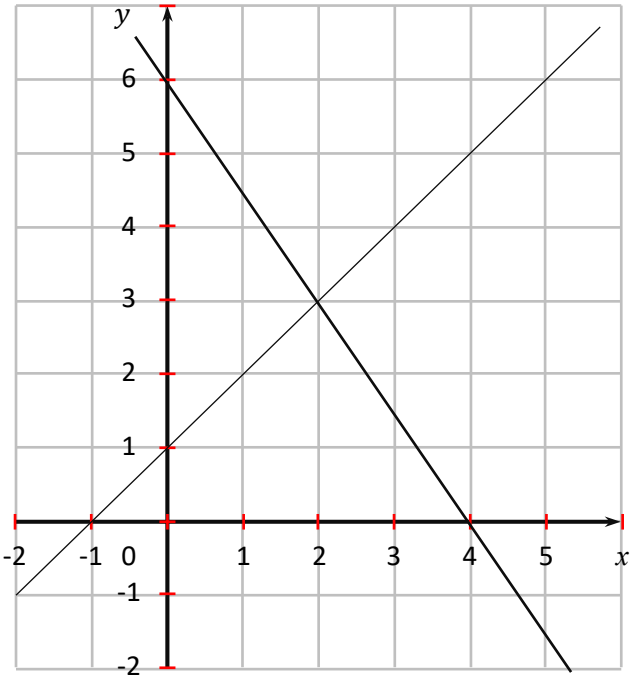
$= 2\frac{25}{28}$ (1)

……

(3)

(Total for Question 9 is 3 marks)

10.



The diagram shows two straight lines.

The equation of the lines are $y = x + 1$ and $3x + 2y = 12$.

a. Write down the solutions of the simultaneous equations.

$$y = x + 1$$

$$3x + 2y = 12$$

The lines intersect at the point (2,3), so the solutions are $x = 2$ and $y = 3$ (1)

.....

(1)

b. Find an equation of the line which is perpendicular to the line with equation $3x + 2y = 12$ and passes through the point (0,9).

Putting $3x + 2y = 12$ into the form of $y = mx + c$:

$y = -\frac{3}{2}x + \frac{12}{2}$ (1)

Gradient of the line: $m = -\frac{3}{2}$ (1)

The gradient of the perpendicular line will be the negative reciprocal of the given gradient

$M = \frac{2}{3}$ (1)

$y = \frac{2}{3}x + c$ (1)

To calculate c, substitute in coordinates that we know will be on the line, in this case (0,9)

$9 = \frac{2}{3}(0) + c$, so $c = 9$

Equation of line: $y = \frac{2}{3}x + 9$

.....

(4)

(Total for Question 10 is 5 marks)

11. There are 11 players in a football team A.

Here are the ages, in years, of these players.

20 25 17 28 19 25 24 18 27 22 20

a. i. Find the interquartile range of these ages.

17 18 19 20 20 22 24 25 25 27 28

LQ = 3rd 19 UQ = 9th 25 (1) both values IQR = 25-29 = 6 (1)

……

(2)

ii. Find the median of these ages.

Median = 5.5 th so 6th value 22 (1)

……

(1)

Here is the information about the ages, of the 11 players in a football team B.

Median	26
Lower quartile	20
Upper quartile	31
Least age	19
Greatest age	35

Use the information above to compare the ages of football team A and team B.

Write down two comparisons.

The ages of the players in football team A are less spread out. (1)

The players of football team A are younger than players of football team B (1)

……

……

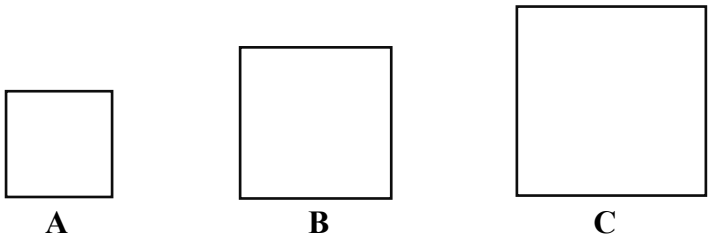
……

……

(2)

(Total for Question 11 is 5 marks)

12. Here are the three squares



The area of square B is 40% more than the area of square A.

The area of square C is 60% more than the area of square B.

Find the area of square A as a fraction of the area of square C.

$area_B = \frac{140}{100} area_A$ $area_C = \frac{160}{100} area_B = \frac{140\times160}{100\times100} area_A$

any one correct (1)

$\frac{A_A}{A_C} = \frac{A_A}{\frac{160}{100}\times\frac{140}{100}\times A_A}$ (1)

$= \frac{1}{\frac{56}{25}}$

$= \frac{25}{56}$ (1)

.....

(Total for Question 12 is 3 marks)

13. Given that n can be any integer such that $n > 1$, prove that $n + n^2$ is never an odd number.

$2 + 2^2 = 6$

$3 + 3^2 = 12$

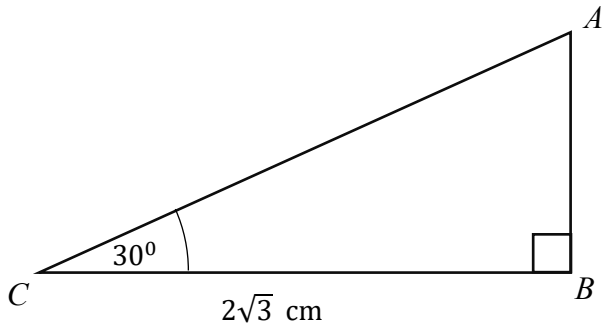
$4 + 4^2 = 20$

$5 + 5^2 = 30$ (1) giving at least three examples

So $n + n^2$ is never an odd number (1) conclusion

(Total for Question 13 is 2 marks)

14.



Triangle ABC is right-angled at B.

$BC = 2\sqrt{3}$ cm and $C = 30^0$

Work out the length of AB.

$\tan 30^0 = \frac{AB}{2\sqrt{3}}$ (1)

$AB = 2\sqrt{3}\tan 30^0$ (1)

$AB = 2$ (1)

.....

(Total for Question 14 is 3 marks)

15. Given that $6x^2 - 13x - 5 = 0$

Find the possible values of x.

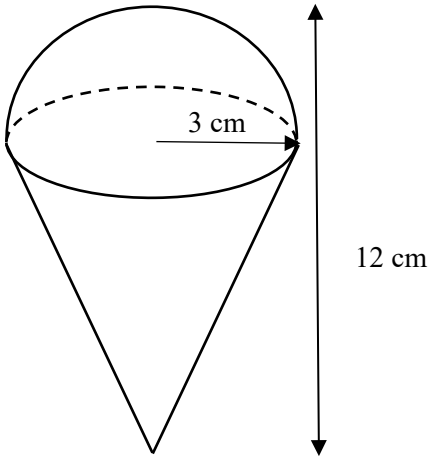
$(3x + 1)(2x - 5) = 0$ (1)

$x = -\frac{1}{3}$ $x = \frac{5}{2}$ (1) at least one correct (1) both correct

.....

(Total for Question 15 is 3 marks)

16. The diagram shows a solid shape made from a hemisphere and a cone.



The hemisphere and a cone both have a radius of 3 cm.

The total height of the solid shape is 12 cm.

Calculate the volume of the solid, in terms of π.

Height of cone $12cm - 3cm = 9cm$

$V_{cone} = \frac{1}{3} \times \pi \times 3^2 \times 9 = 27\pi$

$V_{hemisphere} = \frac{1}{2} \times \pi \times 3^3 = 18\pi$

$V_{total} = 27\pi + 18\pi$

$= 45\pi$

.....

(Total for Question 16 is 4 marks)

17. Express $\frac{(2\sqrt{3})^2}{6-3\sqrt{3}}$ in the form $p(q+\sqrt{3})$ where p and q are integers.

$$\frac{2\sqrt{3}\times 2\sqrt{3}}{6-3\sqrt{3}} = \frac{4\times 3}{6-3\sqrt{3}}$$

$$= \frac{12}{6-3\sqrt{3}}$$

$$= \frac{12(6+3\sqrt{3})}{(6-3\sqrt{3})(6+3\sqrt{3})}$$

$$= \frac{72+36\sqrt{3}}{(36+18\sqrt{3}-18\sqrt{3}-27)}$$

$$= \frac{72+36\sqrt{3}}{9}$$

$$= 8+4\sqrt{3}$$

$$= 4(2+\sqrt{3})$$

.....

(Total for Question 17 is 3 marks)

18. Given that $x^2+8x+10 = (x+a)^2+b$ for all values of x,

a. find the value of a and the value of b.

$$(x+4)^2-4^2+10$$

$$(x+4)^2-6$$

.....

(2)

b. Write down the minimum value of the expression $x^2+8x+10$

-6

.....

(1)

c. Write down the equation of the line of symmetry of the graph of $y = x^2+8x+10$

$$x = -4$$

.....

(1)

(Total for Question 18 is 4 marks)

19. F is directly proportional to d^2

H is inversely proportional to F

Given that $H = 10$ and $d = 2$ when $F = 20$

Find a formula for H in terms of d.

$F = kd^2$

$20 = k2^2$

$k = 5$

$H = \frac{m}{F}$

$10 = \frac{m}{20}$

$m = 200$

$F = 5d^2$

$H = \frac{200}{F}$

$H = \frac{200}{5d^2}$

$H = \frac{40}{d^2}$

.....

(Total for Question 19 is 4 marks)

20. Given the function

$f(x) = 2x - 1$ and $g(x) = x^2$

Show that equation $f^{-1}(x) = gf(x)$ is $8x^2 - 9x + 1 = 0$

$y = 2x - 1$

Swap x and y:

$x = 2y - 1$

Rearrange to find y:

$y = f^{-1}(x) = \frac{x+1}{2}$

Equate with gf(x) and rearrange:

$\frac{x+1}{2} = (2x - 1)^2$

$\frac{x+1}{2} = 4x^2 - 4x + 1$

$x + 1 = 8x^2 - 8x + 2$

$8x^2 - 9x + 1 = 0$

(Total for Question 20 is 5 marks)

21. There are only red discs and black discs in a bag.

There are two less red discs than black discs in a bag.

The probability that Arthur will take two discs of different colour is $\frac{15}{28}$.

Work out the number of black discs in the bag.

Number of black discs=x Number of red discs=$x-2$ Total number of discs=$2x-2$

$$\frac{x-2}{2x-2}\times\frac{x}{2x-3}+\frac{x}{2x-2}\times\frac{x-2}{2x-3}=\frac{15}{28}$$

$$\frac{2x(x-2)}{(2x-2)(2x-3)}=\frac{15}{28}$$

$$56x(x-2)=15(2x-2)(2x-3)$$

$$2x^2-19x+45=0$$

$$(2x-9)(x-5)=0$$

$$x=4.5 \qquad x=5$$

x must be a whole number so black discs = 5

……..

(Total for Question 21 is 5 marks)