

Write your name here

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MODEL SOLUTIONS

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**Pearson Edexcel
Level 3 GCE**

Centre Number

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Candidate Number

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Mathematics

Advanced**Paper 2: Pure Mathematics 2**

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference

9MA0/02**You must have:**

Mathematical Formulae and Statistical Tables, calculator

Total Marks

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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a .

(3)

If $(x+a)$ is a factor, $f(-a) = 0$

$(x+2)$ is a factor, $f(-2) = 0$

$$2(-2)^3 - 5(-2)^2 + a(-2) + a = 0 \quad - (1)$$

$$-16 - 20 - 2a + a = 0$$

$$-36 - a = 0 \quad - (1)$$

$$a = -36 \quad - (1)$$

(Total for Question 1 is 3 marks)

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2. Some A level students were given the following question.

Solve, for $-90^\circ < \theta < 90^\circ$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

<u>Student A</u>
$\cos \theta = 2 \sin \theta$
$\tan \theta = 2$
$\theta = 63.4^\circ$

<u>Student B</u>
$\cos \theta = 2 \sin \theta$
$\cos^2 \theta = 4 \sin^2 \theta$
$1 - \sin^2 \theta = 4 \sin^2 \theta$
$\sin^2 \theta = \frac{1}{5}$
$\sin \theta = \pm \frac{1}{\sqrt{5}}$
$\theta = \pm 26.6^\circ$

$$\sin^2 \theta + \cos^2 \theta = 1$$

- (a) Identify an error made by student A.

(1)

Student B gives $\theta = -26.6^\circ$ as one of the answers to $\cos \theta = 2 \sin \theta$.

- (b) (i) Explain why this answer is incorrect.

- (ii) Explain how this incorrect answer arose.

(2)

a) $\frac{\cos \theta}{\sin \theta} = \tan \theta$

It should be $\frac{\sin \theta}{\cos \theta} = \tan \theta$

b) i) $\cos(-26.6) = 0.894$

$2 \sin(-26.6) = -0.896$

$\cos \theta \neq 2 \sin \theta$

\therefore answer is incorrect.

ii) the incorrect answer arose from squaring.

(Total for Question 2 is 3 marks)

3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n , A and B are constants to be found.

(4)

If $y = uv$, $\frac{dy}{dx} = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$$u = x \quad v = (2x + 1)^4$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = 4(2x + 1)^3 \times 2 \quad - (1)$$

$$= 8(2x + 1)^3$$

$$x \times 8(2x + 1)^3 + (2x + 1)^4 \times 1$$

$$= 8x(2x + 1)^3 + (2x + 1)^4 \quad - (1)$$

$$= (2x + 1)^3 (8x + 2x + 1) \quad - (1)$$

$$= (2x + 1)^3 (10x + 1)$$

$$n = 3$$

$$A = 10$$

$$B = 1$$

- (1)

(Total for Question 3 is 4 marks)

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4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3 \ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

$$a) gf(x) = g(f(x)) = g(e^x)$$

$$\ln x^n = n \ln x$$

$$\ln e = 1$$

$$g(e^x) = 3 \ln e^x \quad - (1)$$

$$= 3x \ln e$$

$$= 3x \times 1$$

$$= 3x$$

$$gf(x) = 3x \quad - (1)$$

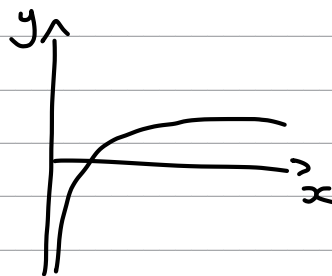
$$b) fg(x) = f(g(x)) = f(3 \ln x)$$

$$e^{\ln x} = x$$

$$f(3 \ln x) = e^{3 \ln x}$$

$$= e^{\ln x^3}$$

$$= x^3$$



$$3x = x^3 \quad - (1)$$

$$x^2 = 3$$

$$x = \pm \sqrt{3} \quad - (1)$$

$y = \ln x$ isn't defined when $x \leq 0$

If $y = \ln x$ isn't defined at $x \leq 0$, $x = \sqrt{3}$ and so there's one real value. $- (1)$

(Total for Question 4 is 5 marks)

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

- (a) find the mass of the radioactive substance six months after it was first observed, (2)

- (b) show that $\frac{dm}{dt} = km$, where k is a constant to be found. (2)

a) 6 months = 0.5 years
at $t = 0.5$ - (1)

$$m = 25e^{-0.05 \times 0.5}$$

$$m = 24.4 \text{ g (3 s.f.)} - (1)$$

b) $y = e^{kx}$
 $\frac{dy}{dx} = ke^{kx}$

$$\frac{dm}{dt} = -0.05 \times 25e^{-0.05t} - (1)$$

$$= -0.05m$$

$$\therefore k = -0.05 - (1)$$

(Total for Question 5 is 4 marks)

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6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$) has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive.	✓			$x^2 - 6x + 10$ $(x - 3)^2 - 9 + 10$ $(x - 3)^2 + 1$ $(x - 3)^2 \geq 0$ $(x - 3)^2 + 1 \geq 1$
(ii) If $ax > b$ then $x > \frac{b}{a}$		✓		If $a > 0$ If $a < 0$ $ax > b$ $ax > b$ $x > \frac{b}{a}$ $x < \frac{b}{a}$
(iii) The difference between consecutive square numbers is odd.	✓			$n^2, (n+1)^2$ $(n+1)^2 - n^2$ $n^2 + 2n + 1 - n^2$ $2n + 1$ $2n + 1$ is the expression for odd numbers.

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x , to show that

$$\sqrt{4-x} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

- (b) State, giving a reason, if the expansion is valid for this value of x .

(1)

$$\begin{aligned} \text{a) } \sqrt{4-x} &= (4-x)^{\frac{1}{2}} \quad - \textcircled{1} \\ &= 4^{\frac{1}{2}} \left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} \\ &= 2 \left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} \quad \checkmark \\ &= 2 \left(1 + \frac{1}{2} \left(-\frac{1}{4}x\right) + \frac{\frac{1}{2}x - \frac{1}{2}}{2!} \left(-\frac{1}{4}x\right)^2 + \dots\right) \quad - \textcircled{1} \\ &= 2 \left(1 - \frac{1}{8}x - \frac{1}{128}x^2 + \dots\right) \quad - \textcircled{1} \\ &= 2 - \frac{1}{4}x - \frac{1}{64}x^2 + \dots \\ \therefore k &= -\frac{1}{64} \quad - \textcircled{1} \end{aligned}$$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots$

$$\text{b) } (1+x)^n \Rightarrow \text{valid for } |x| < 1$$

$$\left(1 - \frac{1}{4}x\right)^{\frac{1}{2}} \Rightarrow \text{valid for } \left| -\frac{1}{4}x \right| < 1$$

$$\left| -\frac{1}{4}x \right| < 1$$

$$|x| < 4$$

$$|x| < 4$$

The expansion is valid, as $x = 1$ fits into $|x| < 4$

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8.

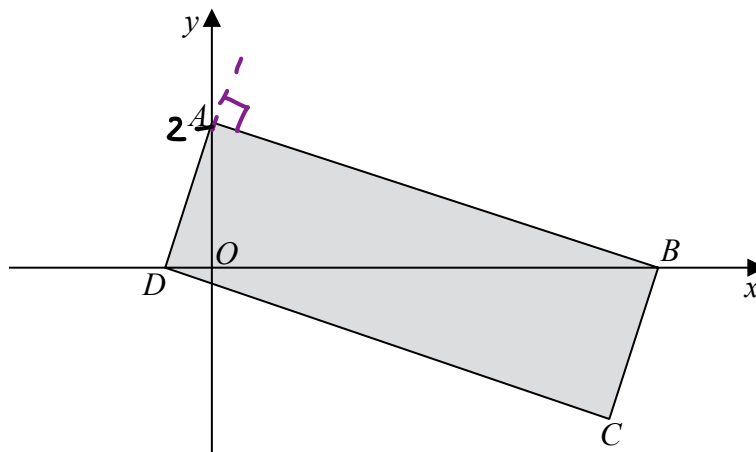


Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y -axis and the points B and D lie on the x -axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$ (4)

(b) find the area of the rectangle $ABCD$. (3)

$$\begin{aligned} \text{a) } 5y + 2x &= 10 \\ y &= -\frac{2}{5}x + 2 \end{aligned}$$

$$m_{AB} = -\frac{2}{5} \quad - \textcircled{1}$$

$$\begin{aligned} -\frac{2}{5} \times m_{AD} &= -1 \\ m_{AD} &= \frac{5}{2} \quad - \textcircled{1} \end{aligned}$$

When $x = 0$:

$$\begin{aligned} 5y + 2(0) &= 10 \\ 5y &= 10 \\ y &= 2 \end{aligned}$$

$$A(0, 2) \quad - \textcircled{1}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{5}{2}(x - 0)$$

$$y - 2 = \frac{5}{2}x$$

$$2y - 4 = 5x$$

$$2y - 5x = 4 \quad - \textcircled{1}$$

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Question 8 continued

b) for B:
When $y=0$;

$$2x = 10$$

$$x = 5$$

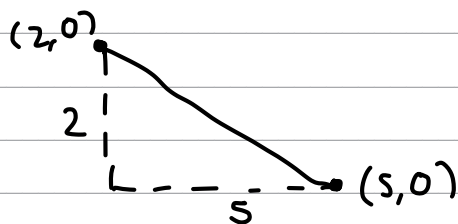
$$B(5, 0)$$

for D:
When $y=0$;

$$-5x = 4$$

$$x = -\frac{4}{5}$$

$$D(-\frac{4}{5}, 0)$$



$$AD = \sqrt{\left(-\frac{4}{5}\right)^2 + 2^2}$$

$$= \frac{2\sqrt{29}}{5}$$

$$AB = \sqrt{2^2 + 5^2}$$

$$= \sqrt{29}$$

- ①

$$\text{Area} = \sqrt{29} \times \frac{2\sqrt{29}}{5} \quad - \text{①}$$

$$= 11.6 \quad - \text{①}$$

(Total for Question 8 is 7 marks)

9. Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A) dx = 2A^2$$

show that there are exactly two possible values for A .

(5)

$$\int_1^4 (3x^{\frac{1}{2}} + A) dx$$

$$= [2x^{\frac{3}{2}} + Ax]_1^4 \quad - (2)$$

$$= [2(4)^{\frac{3}{2}} + A(4)] - [2(1)^{\frac{3}{2}} + A(1)]$$

$$= 16 + 4A - 2 - A$$

$$= 14 + 3A$$

$$14 + 3A = 2A^2 \quad - (1)$$

$$2A^2 - 3A - 14 = 0$$

$$2A^2 + 4A - 7A - 14 = 0$$

$$2A(A+2) - 7(A+2) = 0$$

$$(A+2)(2A-7) = 0 \quad - (1)$$

$$\downarrow$$

$$\downarrow$$

$$A+2=0$$

$$2A-7=0$$

$$A=-2$$

$$A = \frac{7}{2} \quad - (1)$$

(Total for Question 9 is 5 marks)

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10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_{\infty} = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

$$S_{\infty} = \frac{a}{1-r}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$\frac{a}{1-r} = \frac{8}{7} \left(\frac{a(1-r^6)}{1-r} \right)$$

$$\frac{a}{1-r} = \frac{8a(1-r^6)}{7(1-r)}$$

$$a = \frac{8a(1-r^6)}{7}$$

$$1 = \frac{8(1-r^6)}{7}$$

$$1-r^6 = \frac{8}{7}$$

$$r^6 = \frac{1}{7}$$

$$r = \sqrt[6]{\frac{1}{7}}$$

$$r = \frac{1}{\sqrt{2}}$$

$$r = \pm \frac{1}{\sqrt{2}}$$

$$\therefore k = 2$$

(Total for Question 10 is 4 marks)

11.

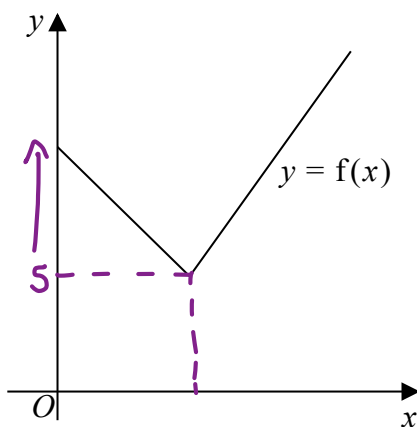


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k .

(2)

a) range: $f(x) \geq 5$

b) $2|3-x| + 5 = \frac{1}{2}x + 30$

$$\begin{array}{l}
 -2(3-x) + 5 = \frac{1}{2}x + 30 \quad - \textcircled{1} \\
 -6 + 2x + 5 = \frac{1}{2}x + 30 \\
 2x - 1 = \frac{1}{2}x + 30 \\
 \frac{3}{2}x = 31 \quad - \textcircled{1} \\
 x = \frac{62}{3}
 \end{array}
 \qquad
 \begin{array}{l}
 2(3-x) + 5 = \frac{1}{2}x + 30 \\
 6 - 2x + 5 = \frac{1}{2}x + 30 \\
 11 - 2x = \frac{1}{2}x + 30 \\
 \frac{5}{2}x = -19 \\
 x = -\frac{38}{5} \quad x
 \end{array}$$

$x = \frac{62}{3} \quad - \textcircled{1}$

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Question 11 continued

$$c) k > 5 - \textcircled{1}$$

When $x=0$;

$$2(3-0) + 5 = y$$

$$2(3) + 5 = y$$

$$y = 11$$

$$k \leq 11$$

$$\{k: k \in \mathbb{R}, 5 < k \leq 11\} - \textcircled{1}$$

(Total for Question 11 is 6 marks)

12. (a) Solve, for $-180^\circ \leq x < 180^\circ$, the equation

$$3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3 \sin^2(2\theta - 30^\circ) + \sin(2\theta - 30^\circ) + 8 = 9 \cos^2(2\theta - 30^\circ)$$

giving your answer to 2 decimal places.

(2)

a) $\sin^2 x + \cos^2 x \equiv 1$
 $\cos^2 x \equiv 1 - \sin^2 x$

$$3 \sin^2 x + \sin x + 8 = 9(1 - \sin^2 x) \quad - (1)$$

$$3 \sin^2 x + \sin x + 8 = 9 - 9 \sin^2 x$$

$$12 \sin^2 x + \sin x - 1 = 0 \quad - (1)$$

$$12 \sin^2 x + 4 \sin x - 3 \sin x - 1 = 0$$

$$4 \sin x (3 \sin x + 1) - (3 \sin x + 1) = 0$$

$$(3 \sin x + 1)(4 \sin x - 1) = 0 \quad - (1)$$

$$\downarrow$$

$$\downarrow$$

$$3 \sin x + 1 = 0$$

$$4 \sin x - 1 = 0$$

$$\sin x = -\frac{1}{3} \quad - (1) \quad - \quad \sin x = \frac{1}{4}$$

$$x = -19.47^\circ, -160.53^\circ \quad x = 14.48^\circ, 165.52^\circ \quad - (1)$$

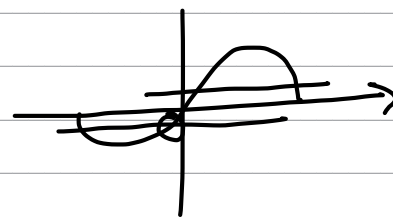
$$x = -160.53^\circ, -19.47^\circ, 14.48^\circ, 165.52^\circ \quad (2 \text{ d.p.}) \quad - (1)$$

b) $2\theta - 30^\circ = -160.53^\circ \quad x$

$$2\theta - 30^\circ = -19.47^\circ \quad - (1)$$

$$2\theta = 10.53^\circ$$

$$\theta = 5.26^\circ \quad - (1)$$



13. (a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$. Give the exact value of R and give the value of α , in degrees, to 2 decimal places. (3)

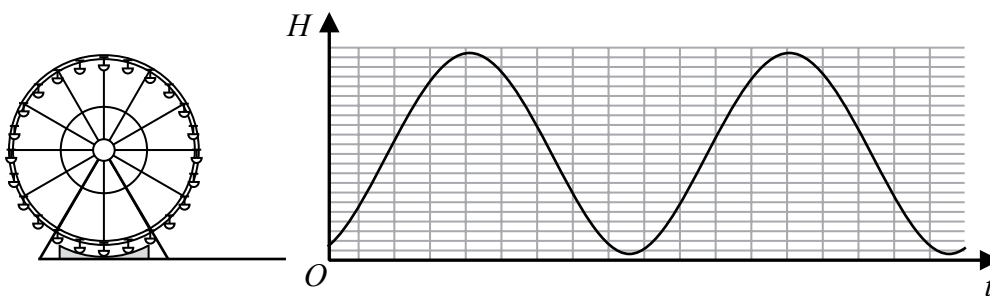


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

- (b) (i) find a complete equation for the model, (ii) hence find the maximum height of the passenger above the ground. (2)
- (c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle. (3)

(Solutions based entirely on graphical or numerical methods are not acceptable.)

It is decided that, to increase profits, the speed of the wheel is to be increased.

- (d) How would you adapt the equation of the model to reflect this increase in speed? (1)

a) $R \cos(\theta + \alpha) \equiv R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$

$10 \cos \theta - 3 \sin \theta = R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$

$10 \cos \theta = R \cos \theta \cos \alpha$

$-3 \sin \theta = -R \sin \theta \sin \alpha$

$R \cos \alpha = 10$ - ①

$R \sin \alpha = 3$ - ②

$R^2 \cos^2 \alpha = 100$

$R^2 \sin^2 \alpha = 9$

$R^2 \cos^2 \alpha + R^2 \sin^2 \alpha = 100 + 9$

$\cos^2 \alpha + \sin^2 \alpha \equiv 1$

$R^2 (\cos^2 \alpha + \sin^2 \alpha) = 109$

$R^2 = 109$

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Question 13 continued

$$a) R = \sqrt{109} \quad - (1)$$

$$(2) \div (1)$$

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{3}{10}$$

$$\tan \alpha = \frac{3}{10} \quad - (1)$$

$$\alpha = 16.70^\circ \text{ (2d.p.)}$$

- (1)

$$= \sqrt{109} \cos(\theta + 16.70^\circ)$$

$$b) i) 1 = a - 10 \cos(0) + 3 \sin(0)$$

$$1 = a - 10$$

$$a = 11$$

$$H = 11 - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$$

$$10 \cos \theta - 3 \sin \theta$$

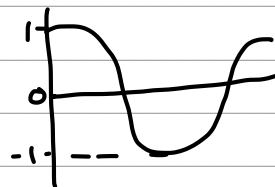
$$= \sqrt{109} \cos(\theta + 16.70^\circ)$$

$$- 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$$

$$= - (10 \cos(80t)^\circ - 3 \sin(80t)^\circ)$$

$$= - \sqrt{109} \cos(80t + 16.70)^\circ$$

$$H = 11 - \sqrt{109} \cos(80t + 16.70)^\circ \quad - (1)$$



$$ii) \cos(80t + 16.70)^\circ = -1$$

$$H = 11 - (\sqrt{109} \times -1)$$

$$H = 11 + \sqrt{109} \quad - (1)$$

$$c) \cos(80t + 16.70)^\circ = -1$$

$$80t + 16.7 = 180 \quad \leftarrow \text{for first cycle}$$

$$180 + 360 = 540$$

$$80t + 16.70 = 540 \quad - (1)$$

$$t = \frac{540 - 16.7}{80} \quad - (1)$$

for second cycle

$$0.54 \times 60$$

$$= 32 \text{ (2s.f.)}$$

(Total for Question 13 is 9 marks)

Question 13 continued

c) $t = 6.54$ minutes

$t = 6$ minutes 32 seconds - (1)

d) $H = 11 - \sqrt{109} \cos(80t + 16.70)^\circ$

We would increase the '80t' value - for example
to 90t. - (1)

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

Volume
= 500 ml

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

- (a) Prove that the total surface area, S cm², of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r} \quad (3)$$

Given that r can vary,

- (b) find the dimensions of a can that has **minimum** surface area.

(5)

- (c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

(1)

a) $SA = 2\pi r^2 + 2\pi rh$
 $V = \pi r^2 h$

$$\pi r^2 h = 500 \quad \text{--- (1)}$$

$$h = \frac{500}{\pi r^2}$$

$$S = 2\pi r^2 + 2\pi r \times \left(\frac{500}{\pi r^2}\right) \quad \text{--- (1)}$$

$$S = 2\pi r^2 + \frac{1000\pi r}{\pi r^2}$$

$$S = 2\pi r^2 + \frac{1000}{r} \quad \text{--- (1)}$$

b) $S = 2\pi r^2 + 1000r^{-1}$

$$\frac{dS}{dr} = 4\pi r - 1000r^{-2}$$

$$= 4\pi r - \frac{1000}{r^2} \quad \text{--- (2)}$$

$$\text{At } \frac{dS}{dr} = 0 \quad \text{--- (1)}$$

$$4\pi r - \frac{1000}{r^2} = 0$$

$$4\pi r^3 - 1000 = 0$$

$$r^3 = \frac{1000}{4\pi}$$

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Question 14 continued

$$b) r = 4.30 \text{ cm (3 s.f.)} - \textcircled{1}$$

$$h = \frac{500}{\pi r^2}$$

$$h = \frac{500}{\pi (4.30)^2}$$

$$h = 8.60 \text{ cm (3 s.f.)} - \textcircled{1}$$

$$\text{Radius} = 4.30 \text{ cm (3 s.f.)}$$

$$\text{Height} = 8.60 \text{ cm (3 s.f.)}$$

$$c) r = 4.30 \text{ cm} \quad h = 8.60 \text{ cm}$$

If the radius is 4.30 cm and the height is 8.60 cm, then the can is square in profile - but all cans are taller than they are wide - $\textcircled{1}$

(Total for Question 14 is 9 marks)

15.

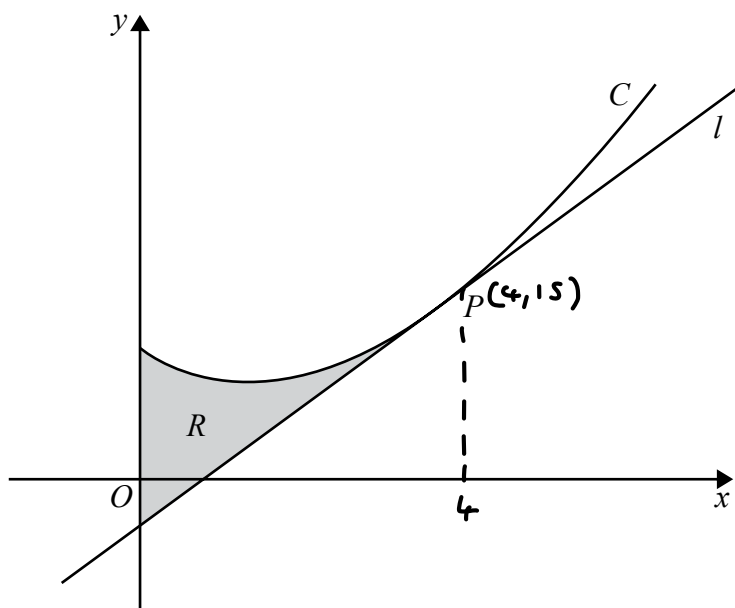


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geq 0$$

The point P with coordinates $(4, 15)$ lies on C .

The line l is the tangent to C at the point P .

The region R , shown shaded in Figure 4, is bounded by the curve C , the line l and the y -axis.

Show that the area of R is 24, making your method clear.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

$$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9 \quad \text{--- (1)}$$

$$y - y_1 = m(x - x_1)$$

When $x = 4$;

$$\frac{15}{2}(4)^{\frac{1}{2}} - 9 = 6 \quad \text{--- (1)}$$

$$y - 15 = 6(x - 4)$$

$$y - 15 = 6x - 24$$

$$y = 6x - 9 \quad \text{--- (2)}$$

$$\int_0^4 (5x^{\frac{3}{2}} - 9x + 11) dx - \int_0^4 (6x - 9) dx \quad \text{--- (1)}$$

$$= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x \right]_0^4 - \left[3x^2 - 9x \right]_0^4 \quad \text{--- (1)}$$

$$= \left(\left[2(4)^{\frac{5}{2}} - \frac{9}{2}(4)^2 + 11(4) \right] - 0 \right) - \left(\left[3(4)^2 - 9(4) \right] - 0 \right) \quad \text{--- (1)}$$

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Question 15 continued

$$= 36 - 12$$

$$= 24$$

$$\text{Area of } R = 24 - \textcircled{1}$$

(Total for Question 15 is 10 marks)

16. (a) Express $\frac{1}{P(11-2P)}$ in partial fractions. (3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began, $t=0, P=1$

(b) determine the time taken, in years, for this population of meerkats to double, (6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A, B and C are integers to be found. (3)

a) $\frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$ - (1)

$$1 = A(11-2P) + BP$$

When $P=0$; When $P=\frac{11}{2}$;
 $1 = 11A$ $1 = \frac{11}{2}B$ - (1)
 $A = \frac{1}{11}$ $B = \frac{2}{11}$

$$= \frac{\frac{1}{11}}{P} + \frac{\frac{2}{11}}{11-2P} \quad (1) \quad \text{or} \quad \frac{1}{11P} + \frac{2}{11(11-2P)}$$

b) $\frac{dP}{dt} = \frac{1}{22}P(11-2P)$

$$\int \frac{22}{P(11-2P)} \cdot dP = \int dt \quad - (1)$$

$$\int \left(\frac{2}{P} + \frac{4}{11-2P} \right) dP = \int dt \quad - (1)$$

$$2 \ln P - 2 \ln(11-2P) = t + C \quad - (1)$$

$$\frac{1}{P(11-2P)} = \frac{\frac{1}{11}}{P} + \frac{\frac{2}{11}}{11-2P}$$

$$\frac{22}{P(11-2P)} = \frac{2}{P} + \frac{4}{11-2P}$$

$$\int \frac{1}{x} \cdot dx = \ln|x|$$

$$\frac{4 \ln(11-2P)}{-2} = -2 \ln(11-2P)$$

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Question 16 continued

b) When $t=0$, $P=1$;

$$2\ln 1 - 2\ln 9 = C$$

$$C = -2\ln 9 \quad - (1)$$

$$2\ln \frac{1}{9} = 2\ln 9^{-1} \\ = -2\ln 9$$

$$2\ln P - 2\ln(11-2P) = t - 2\ln 9$$

$$\text{At } P=2; \quad - (1)$$

$$2\ln 2 - 2\ln 7 = t - 2\ln 9$$

$$t = 1.89 \text{ years (3 s.f.)} \quad - (1)$$

c) $2\ln P - 2\ln(11-2P) = t - 2\ln 9$

$$2\ln\left(\frac{P}{11-2P}\right) = t - 2\ln 9$$

$$2\ln\left(\frac{9P}{11-2P}\right) = t$$

$$\ln\left(\frac{9P}{11-2P}\right) = \frac{1}{2}t \quad - (1)$$

$$\frac{9P}{11-2P} = e^{\frac{1}{2}t}$$

$$9P = 11e^{\frac{1}{2}t} - 2Pe^{\frac{1}{2}t} \quad - (1)$$

$$P(9 + 2e^{\frac{1}{2}t}) = 11e^{\frac{1}{2}t}$$

$$P = \frac{11e^{\frac{1}{2}t}}{9 + 2e^{\frac{1}{2}t}} \times \frac{e^{-\frac{1}{2}t}}{e^{-\frac{1}{2}t}}$$

$$P = \frac{11}{9e^{-\frac{1}{2}t} + 2}$$

$$P = \frac{11}{2 + 9e^{-\frac{1}{2}t}}$$

$$A = 11$$

$$B = 2 \quad - (1)$$

$$C = 9$$