# Pearson Edexcel Level 3

# GCE Mathematics

## Advanced

## Paper 1: Pure Mathematics

**PMT Mock 2**

| Time: 2 hours | Paper Reference(s) 9MA0/01 |
| --- | --- |

**You must have:**
**Mathematical Formulae and Statistical Tables, calculator**

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions**

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

**Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions. Write your answers in the spaces provided.**

1. Given that $\theta$ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{\theta \tan 3\theta}{\cos 2\theta - 1}$$

For small angles, $\tan\theta \approx \theta, \cos\theta \approx 1 - \frac{\theta^2}{2}$ and substituting in:

$$\frac{\theta \tan 3\theta}{\cos 2\theta - 1} \approx \frac{\theta \times 3\theta}{1 - \frac{(2\theta)^2}{2} - 1}$$

$$\approx \frac{3\theta^2}{-2\theta^2}$$

$$\approx -\frac{3}{2}$$

M1 Attempts to use the small angle approximations

M1 Substitutes correctly and attempts to simplify

A1 Uses both identities and simplifies to $-\frac{3}{2}$

**(Total for Question 1 is 3 marks)**

2.

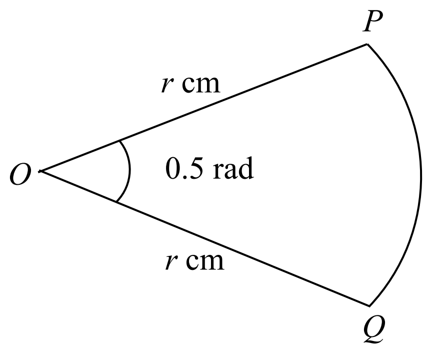



**Figure 1**

Figure 1 shows a sector $POQ$ of a circle with centre $O$ and radius $r$ cm.

The angle $POQ$ is 0.5 radians.

The area of the sector is 9 cm$^2$.

Show that the perimeter of the sector is $k$ times the length of the arc, where $k$ is an integer.

Area of a sector: $\frac{1}{2}r^2\theta = 9$

$\frac{1}{2}r^2(0.5) = 9 \Rightarrow r = 6$

Arc length$= r\theta$

$= 6 \times 0.5 = 3$

Total perimeter: $6 + 6 + 3 = 15 = 5(3) \Rightarrow k = 5$

M1 Attempts to use the area to find a value for $r$

A1 correct value for $r$

M1 Uses arc length formula to attempt to find the value of $k$

A1 Correct solution only

**(Total for Question 2 is 4 marks)**

3. The curve $C$ has equation

$$y = 8\sqrt{x} + \frac{18}{\sqrt{x}} - 20 \qquad x > 0$$

a. Find

i) $\frac{dy}{dx}$

ii) $\frac{d^2y}{dx^2}$

i) M1 Differentiates to $\frac{dy}{dx} = Ax^{-\frac{1}{2}} + Bx^{-\frac{3}{2}}$

A1 Achieves a correct $\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}}$

ii) B1 Achieves a correct $\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}} + \frac{27}{2}x^{-\frac{5}{2}}$ for their $\frac{dy}{dx}$

**(3)**

b. Use calculus to find the coordinates of the stationary point of $C$.

At stationary points, $\frac{dy}{dx} = 0$:

$4x^{-\frac{1}{2}} - 9x^{-\frac{3}{2}} = 0 \Rightarrow 4x - 9 = 0 \Rightarrow x = \frac{9}{4}$

Substituting into $C$:

$$y = 8\sqrt{\frac{9}{4}} + \frac{18}{\sqrt{(9 \div 4)}} - 20 = 4$$

Stationary point at $\left(\frac{9}{4}, 4\right)$

M1 Set $\frac{dy}{dx} = 0$

A1 Rearrange to fine $x$, correct answer only

M1 substitutes their $x$ value(s) into the given equation

A1 correct answer only

**(4)**

c. Determine whether the stationary point is a maximum or minimum, giving a reason for your answer.

$\frac{d^2y}{dx^2} = -2\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{27}{2}\left(\frac{9}{4}\right)^{-\frac{5}{2}}$

$\frac{d^2y}{dx^2} = \frac{32}{27}$

$\frac{d^2y}{dx^2} > 0$, so is a minimm

M1 substitutes their $x$ value(s) into their second derivative

A1 Fully correct solution including a correct numerical second derivative (awrt) and a reference to positive or > 0 and a conclusion

**(2)**

**(Total for Question 3 is 9 marks)**

4. The curve with equation $y = 2 + \ln(4 - x)$ meets the line $y = x$ at a single point, $x = \beta$.

a. Show that $2 < \beta < 3$

$f(x) = 2 + \ln(4 - x) - x$ (as this will mean $f(x) = 0$ at the intersection)

$$f(2) = 2 + \ln(2) - 2 = 0.69315\ldots$$

$$f(3) = 2 + \ln(1) - 3 = -1$$

As $f(x)$ is a continuous function, and there is a sign change in the interval [2,3], this means that $f(x) = 0$ in this interval and therefore $2 + \ln(4 - x) = x$.

M1 Attempts f(2) = … and f(3) = … where $f(x) = \pm(2 + \ln(4 - x) - x)$

A1 Both values correct to at least 1 significant figure with correct explanation and conclusion.

**(2)**

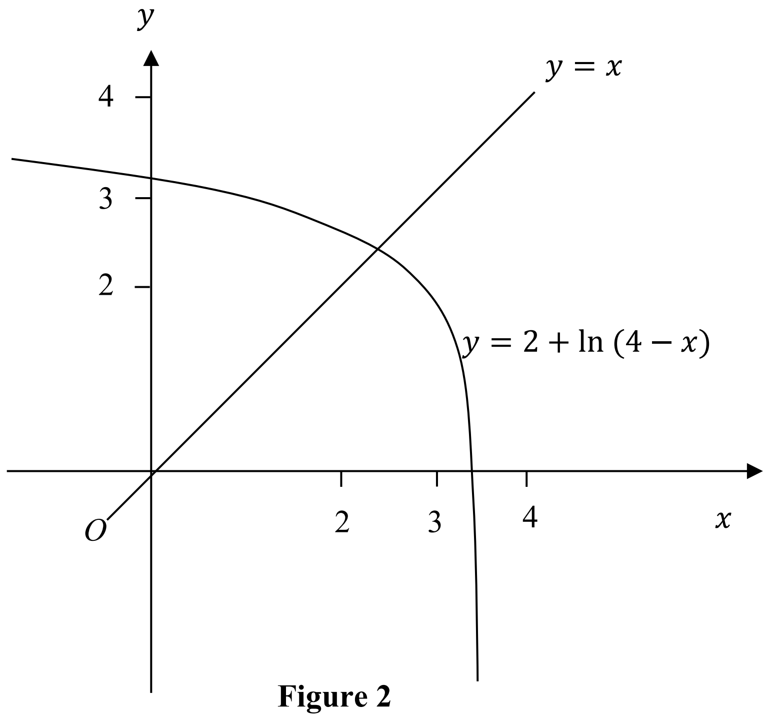



**Figure 2**

Figure 2 shows the graph of $y = 2 + \ln(4 - x)$ and the graph of $y = x$.

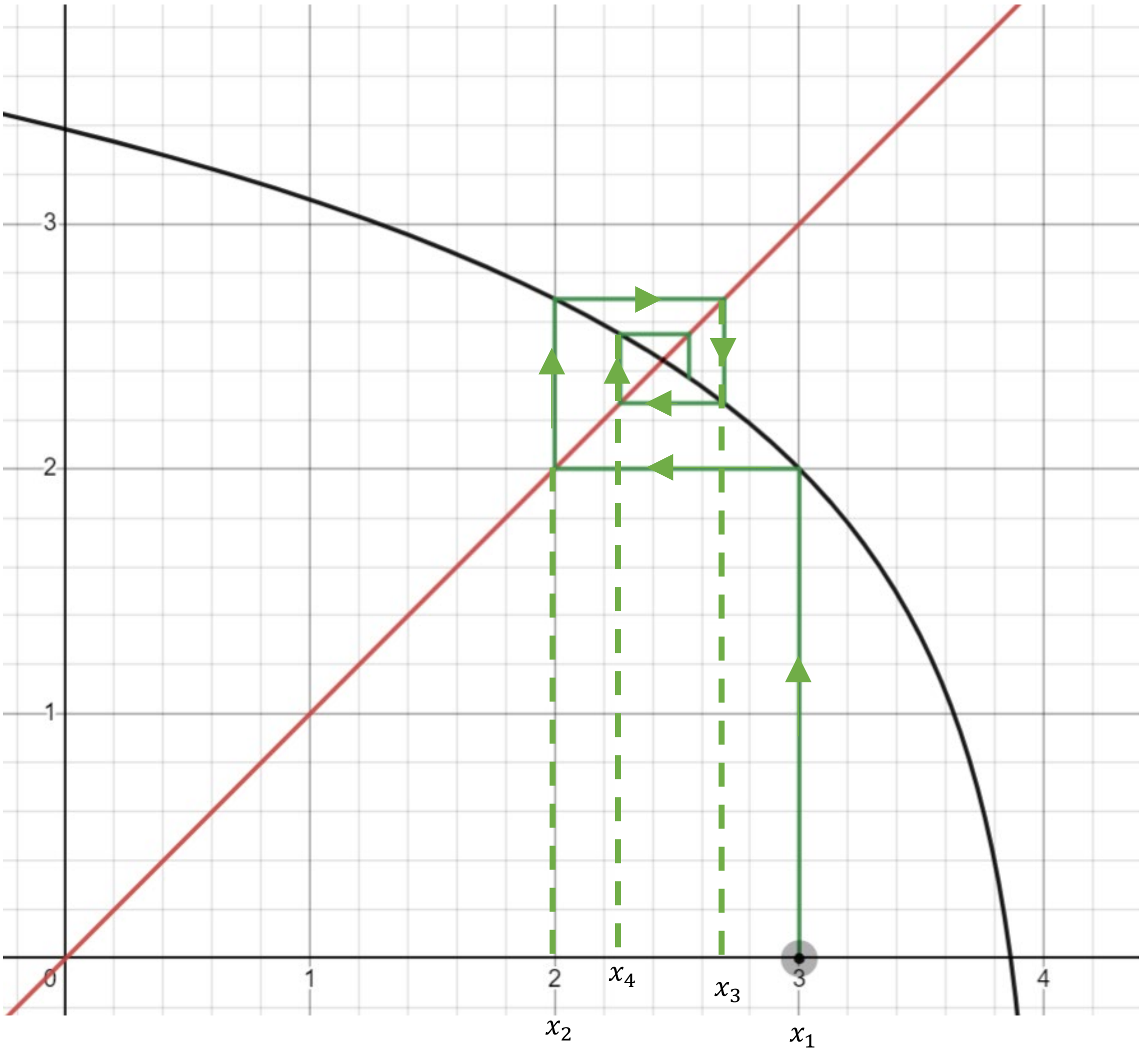
A student uses the iteration formula

$$x_{n+1} = 2 + \ln(4 - x_n), \qquad n \in N,$$

in an attempt to find an approximation for $\beta$.

Using the graph and starting with $x_1 = 3$,

b. determine whether the or not this iteration formula can be used to find an approximation for $\beta$, justifying your answer.




As the cobweb spirals inwards, it converges to the root

M1 For an attempt at using a cobweb diagram starting at $x_1 = 3$

It should have at least two spirals.

A1 For a correct attempt starting at 3 and deducing that the iteration formula can be used to find an approximation for $\beta$ because 'the cobweb spirals inwards' or 'the cobweb gets closer to the root' or 'the cobweb converges'.

**(2)**

**(Total for Question 4 is 4 marks)**

5. Given that

$$y = \frac{5\cos\theta}{4\cos\theta + 4\sin\theta}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

Show that

$$\frac{dy}{d\theta} = -\frac{5}{4(1+\sin 2\theta)}, \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

Applying the quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - \frac{udv}{dx}}{v^2}$

$$\frac{dy}{d\theta} = \frac{(4\cos\theta + 4\sin\theta)(-5\sin\theta) - (5\cos\theta)(-4\sin\theta + 4\cos\theta)}{(4\cos\theta + 4\sin\theta)^2}$$

$$= \frac{-20\cos\theta\sin\theta - 20\sin^2\theta + 20\cos\theta\sin\theta - 20\cos^2\theta}{16(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta)}$$

$$= \frac{-20(\sin^2\theta + \cos^2\theta)}{16(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta)}$$

Using $\sin^2\theta + \cos^2\theta) = 1$:

$$= \frac{-20}{16(1 + 2\cos\theta\sin\theta)}$$

Using $2\sin\theta\cos\theta = \sin 2\theta$:

$$= \frac{-20}{16(1+\sin 2\theta)} = \frac{-5}{4(1+\sin 2\theta)}$$ as required.

M1 For choosing either the quotient, product rule or implicit differentiation and applying it to the given function

Implicit differentiation look for

$$(\pm \cdots \sin\theta \pm \cdots \cos\theta)y + \frac{dy}{d\theta}(4\cos\theta + 4\sin\theta) = \cdots \sin\theta$$

A1 A correct expression involving $\frac{dy}{d\theta} = \frac{(4\cos\theta + 4\sin\theta) \times -5\sin\theta - 5\cos\theta(-4\sin\theta + 4\cos\theta)}{(4\cos\theta + 4\sin\theta)^2}$

M1 Expands and uses $\sin^2\theta + \cos^2\theta = 1$ at least once in the numerator or the denominator OR uses $2\sin\theta\cos\theta = \sin 2\theta$ in $\frac{dy}{d\theta} = \frac{\cdots}{\ldots\ldots C\sin\theta\cos\theta}$

M1 Expands and uses $\sin^2\theta + \cos^2\theta = 1$ in the numerator and the denominator AND uses $2\sin\theta\cos\theta = \sin 2\theta$ in the denominator to reach an expression of the form

$$\frac{dy}{d\theta} = \frac{P}{Q + R\sin 2\theta}$$

A1 Fully correct proof with $A = -\frac{5}{4}$ **(Total for Question 5 is 5 marks)**

6.

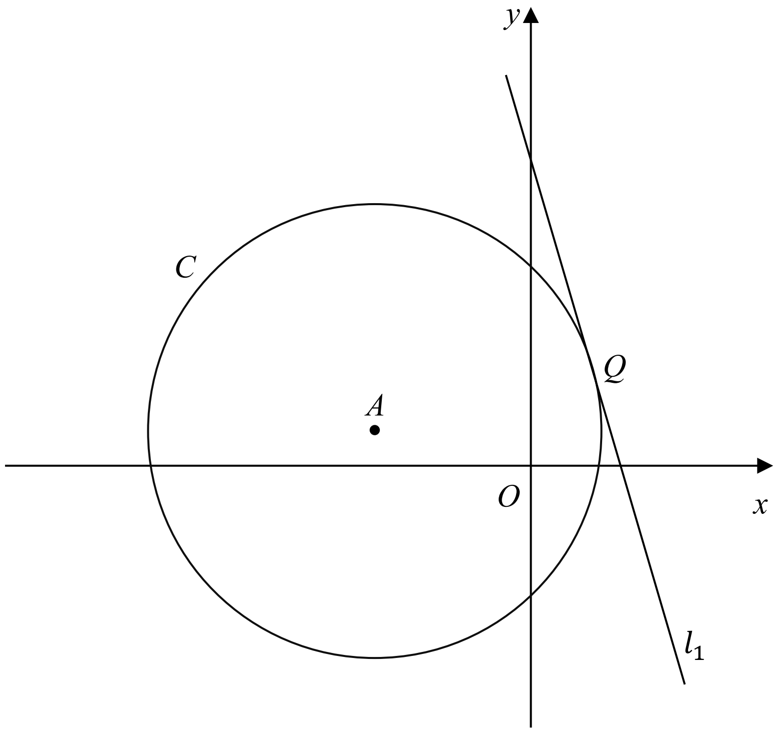



**Figure 3**

The circle $C$ has centre $A$ with coordinates $(-3,1)$.

The line $l_1$ with equation $y = -4x + 6$, is the tangent to $C$ at the point $Q$, as shown in Figure 3.

a. Find the equation of the line $AQ$ in the form $ax + by = c$.

By circle theorems, we know that the line $l_1$ and the radius $AQ$ will be perpendicular, so the gradient of $AQ$ is $\frac{-1}{-4} = \frac{1}{4}$.

Finding equation of a line with gradient $\frac{1}{4}$ that passes through the point $(-3,1)$.

$1 = \frac{1}{4}(-3) + c \Rightarrow c = \frac{7}{4}$

$y = \frac{1}{4}x + \frac{7}{4} \Rightarrow 4y - x = 7$

OR

$y - 1 = \frac{1}{4}(x + 3) \Rightarrow 4y - x = 7$

M1 Correctly deduces the gradient of $AQ$

M1 Finding the equation of a line with gradient $\frac{1}{4}$ and point $(-3,1)$

A1 For expressing the equation of the tangent in the form $4y - x = 7$

**(3)**

b. Show that the equation of the circle $C$ is $(x+3)^2+(y-1)^2=17$

Finding the coordinates of $Q$:

$4y-x=7$ (1)

$y=-4x+6$ (2)

Solving (1) and (2) simultaneously and substituting (2) into (1):

$$4(-4x+6)-x=7 \Rightarrow -17x--17 \Rightarrow x=1$$

Substituting into (2) gives $y=1$, therefore $Q=(1,2)$

Finding the length of $A$ $(-3,1)$ and $Q(2,3)$:

$$r=\sqrt{(1--3)^2+(2-1)^2}=\sqrt{17} \quad \text{or } r^2=17$$

Therefore the equation of the circle is:

$$(x+3)^2+(y-1)^2=17$$

M1 For an attempt at solving $4y-x=7$ and $y=-4x+6$ simultaneously to find the coordinates of point $Q$.

A1 Finds Q correctly

M1 An attempt to use the Pythagoras Theorem to find the radius or (radius)$^2$ using their $Q(1,2)$ and $A(-3,1)$

A1 Correctly states the equation of the circle

**(4)**

The line $l_2$ with equation $y = -4x + k,\ \ k \neq 6$, is also a tangent to $C$.

c. Find the value of the constant $k$.

If $l_2$ is a tangent, then it must intersect the circle once:

$(x+3)^2 + (-4x+k-1)^2 = 17$

$x^2 + 6x + 9 + 16x^2 - 8kx + 8x + k^2 - 2k + 1 = 17$

$17x^2 + (14-8k)x + k^2 - 2k - 7 = 0$

As the tangent intersects the circle once, the discriminant must equal 0:

$b^2 - 4ac = 0$

$(14-8k)^2 - 4(17)(k^2 - 2k - 7) = 0$

$64k^2 - 224k + 196 - 68k^2 + 136k + 476 = 0$

$-8k^2 - 88k + 672 = 0$

$k = 6$ or $k = -28$

The question states $k \neq 6$, so $k = -28$

M1 For solving $l_2$ with their equation of the circle

M1 Uses $b^2 - 4ac = 0$ and finds a value of $k$

A1 $k = -28$

OR

M1 For finding the end point of the diameter using mid-point formula

$$\frac{x+1}{2} = -3 \Rightarrow x = -7 \qquad \frac{y+2}{2} = 1 \Rightarrow y = 0$$

M1 For substituting $(-7,0)$ into $y = -4x + k$ and proceeds to $k = \ldots$

A1 $k = -28$

**(3)**

**(Total for Question 6 is 10 marks)**

7. Given that $k \in \mathbb{Z}^+$

a. show that $\int_{2k}^{3k} \frac{6}{(7k-2x)}\, \mathrm{d}x$ is independent of $k$,

Using integration by substitution:

$u = 7k - 2x \Rightarrow \frac{du}{dx} - 2 \Rightarrow dx = -\frac{du}{2}$

$\int_{2k}^{3k} \frac{6}{(7k-2x)}\, \mathrm{d}x \;= -3\int \frac{1}{u} du = \ln u$

Undoing the substitution:

$$[-3\ln(7k-2x)]_{2k}^{3k} = -3\,(\ln(k) - \ln(3k)) = -3\ln\left(\frac{1}{3}\right)$$

So the integral is independent of $k$.

M1 $\int \frac{6}{(7k-2x)}\, \mathrm{d}x = A\ln(7k-2x)$

A1 $\int \frac{6}{(7k-2x)}\, \mathrm{d}x = -3\ln(7k-2x)$

M1 For substituting $3k$ and $2k$ into their $A\ln(7k-2x)$ and subtracting either way around.

A1 Uses correct ln work and notation to show that $I = -3\ln\left(\frac{1}{3}\right)$ or $\ln 27$

i.e. independent of $k$

**(4)**

b. show that $\int_k^{2k} \frac{2}{3(2x-k)^2}\,\mathrm{d}x$ is inversely proportional to $k$.

Using integration by substitution:

$u = 2x - k \Rightarrow \frac{du}{dx} = 2 \Rightarrow x = \frac{1}{2}$

$\int_k^{2k} \frac{2}{3(2x-k)^2}\,\mathrm{d}x = \frac{1}{3}\int \frac{1}{u^2}\,du$

$= \frac{1}{-3u}$

Undoing substitution:

$\int_k^{2k} \frac{2}{3(2x-k)^2}\,\mathrm{d}x = \left[\frac{-1}{3(2x-k)}\right]_k^{2k}$

$= \frac{-1}{3(3k)} - \frac{-1}{3k} = \frac{2}{9}k$

Therefore, the integral is inversely proportional to $k$ as it is of the form $\frac{C}{k}$ with $C = \frac{2}{9}$

M1 $\int \frac{2}{3(2x-k)^2}\,\mathrm{d}x = \frac{B}{(2x-k)}$

M1 For substituting $2k$ and $k$ into their $\frac{B}{(2x-k)}$ and subtracting

A1 Shows that it is inversely proportional to $k$

**(3)**

**(Total for Question 7 is 7 marks)**

8. The length of the daylight, $D(t)$ in a town in Sweden can be modelled using the equation

$$D(t) = 12 + 9\sin\left(\frac{360t}{365} - 63.435\right) \qquad 0 \leq t \leq 365$$

where $t$ is the number of days into the year and the argument of $\sin x$ is in degrees

a. Find the number of daylight hours after 90 days in that year.

$$D(90) = 12 + 9\sin\left(\frac{360(90)}{365} - 63.435\right)$$

$$D(90) = 15.851 \text{ hours}$$

B1 Finding $D(90) = 15.85 \ldots$ hours

**(1)**

b. Find the values of $t$ when $D(t) = 17$, giving your answers to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*)

$$17 = 12 + 9\sin\left(\frac{360t}{365} - 63.435\right)$$

$$\frac{5}{9} = \sin\left(\frac{360t}{365} - 63.435\right)$$

$$\frac{360t}{365} - 63.435 = 33.74 \text{ or } 146.251$$

$$t = 98.534 \text{ or } 212.59$$

$$t = 99 \text{ or } 213 \text{ to the nearest integer}$$

M1 For using $D = 17$ and proceeding to $\sin\left(\frac{360t}{365} - 63.435\right)^0 = k \qquad |k| \leq 1$

A1 $\sin\left(\frac{360t}{365} - 63.435\right)^0 = \frac{5}{9}$. Using the correct order to find **one correct value** of $t$.

M1 Using the correct order to find a **second value** of $t$

A1 $t = 98.5 = 99$ days and $t = 212.598 = 213$ days

**(4)**

**(Total for Question 8 is 5 marks)**

9.

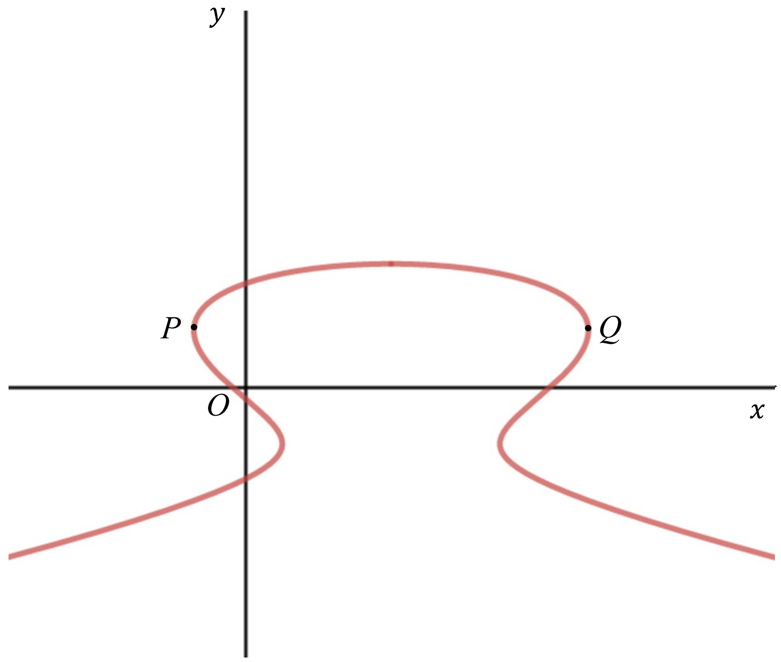



**Figure 4**

Figure 4 shows a sketch of the curve with equation $x^2 + y^3 - 10x - 12y - 5 = 0$

a. Show that $\frac{dy}{dx} = \frac{10-2x}{3y^2-12}$

Using implicit differentiation:

$\frac{d}{dx}(x^2 + y^3 - 10x - 12y - 5) = 2x + 3y^2\frac{dy}{dx} - 10 - 12\frac{dy}{dx} = 0$

$\frac{dy}{dx}(3y^2 - 12) = 10 - 2x$

$\frac{dy}{dx} = \frac{10-2x}{3y^2-12}$

M1 either $y^3 \to Ay^2\frac{dy}{dx}$ or $12y \to 12\frac{dy}{dx}$

A1 Fully correct derivative $2x + 3y^2\frac{dy}{dx} - 10 - 12\frac{dy}{dx} = 0$

M1 For a valid attempt at making $\frac{dy}{dx}$ the subject, with two terms in $\frac{dy}{dx}$ coming from $x^2$ and $2y$.

A1 $\frac{dy}{dx} = \frac{10-2x}{3y^2-12}$ with no errors

**(4)**

At each of the points $P$ and $Q$ the tangent to the curve is parallel to the $y$-axis.

b. Find the exact coordinates of $Q$.

For the curve to be parallel to the $y$-axis, the derivative is infinite, which means

$$3y^2 - 12 = 0 \Rightarrow y = \pm 2$$

From the diagram, we can see that $y$ is positive at $Q$, so $y = 2$

Substituting into the equation of the curve:

$$x^2 + 8 - 10x - 24 - 5 = 0$$
$$x^2 - 10x - 21 = 0$$
$$x = 5 \pm \sqrt{46}$$

From the diagram we can see that the $x$ value must be positive, so $Q = (2, 5 + \sqrt{26})$

M1 Deduces that $3y^2 - 12 = 0$ and proceed to find the value of $y$

A1 $y = 2$

M1 substitute $y = 2$ in $x^2 + y^3 - 10x - 12y - 5 = 0$ and creating a 3 term quadratic and solving

A1 $Q(5 + \sqrt{46}, 2)$

**(4)**

**(Total for Question 9 is 8 marks)**

10. a. Find $\int \frac{1}{30}\cos\frac{\pi}{6}t \quad dt$.

$$\int \frac{1}{30}\cos\frac{\pi}{6}t \quad dt = \frac{1}{30}\int \cos\frac{\pi}{6} = \frac{1}{\mathbf{30}}\sin\frac{\pi}{6} + c$$

M1 $\int \frac{1}{30}\cos\frac{\pi}{6}t \quad dt = A\sin\frac{\pi}{6}t + c$

A1 Correct integration $\frac{1}{5\pi}\sin\frac{\pi}{6}t + c$

**(2)**

The height above ground, $X$ metres, of the passenger on a wooden roller coaster can be modelled by the differential equation

$$\frac{dX}{dt} = \frac{1}{30}X\cos(\frac{\pi}{6}t)$$

where $t$ is the time, in seconds, from the start of the ride.

At time $t = 0$, the passenger is 6 m above the ground.

b. Show that $X = ke^{\frac{1}{5\pi}\sin(\frac{\pi}{6}t)}$ where the value of the constant $k$ should be found.

Separation of variables gives:

$$\int \frac{1}{X}\,dX = \int \frac{1}{30}\cos\frac{\pi}{6}t \; dt$$

$$\ln X = \frac{1}{5\pi}\sin\frac{\pi}{6}t + c$$

$$X = e^{\frac{1}{5\pi}\sin\frac{\pi}{6}t + c}$$

$$X = ke^{\frac{1}{5\pi}\sin(\frac{\pi}{6}t)}$$

Substituting $t = 0, X = 6$

$$6 = ke^0 \Rightarrow k = 6$$

$$X = 6e^{\frac{1}{5\pi}\sin(\frac{\pi}{6}t)}$$

M1 separation of variables

A1 Integrates both sides

M1 Rearranges to have $X = \cdots$

M1 Substitutes $t = 0$, $X = 6$

A1 Finds $k$

**(4)**

c. Show that the maximum height of the passenger above the ground is 6.39 m.

$X$ is maximised when the exponent is big. The maximum value of $\frac{1}{5\pi}\sin\frac{\pi}{6}t = \frac{1}{5\pi}$, so $X = 6e^{\frac{1}{5\pi}} = 6.3944$

B1 shows that the maximum height is 6.39 m.

**(1)**

The passenger reaches the maximum height, for the second time, *T* seconds after the start of the ride.

d. Find the value of *T*.

$$6e^{\frac{1}{5\pi}} = 6e^{\frac{1}{5\pi}\sin\left(\frac{\pi}{6}t\right)}$$
$$\frac{1}{5\pi} = \frac{1}{5\pi}\sin\left(\frac{\pi}{6}t\right)$$
$$1 = \sin\left(\frac{\pi}{6}t\right)$$
$$\frac{\pi}{6}t = \frac{\pi}{2} + 2k\pi, \quad k \in \mathbb{Z}$$

The passenger reaches the maximum height for the second time when $\frac{\pi}{6}t = \frac{5\pi}{2}$
So $t = 15$.

M1 For identifying that it would reach the maximum height for the second time when $\frac{\pi}{6}t = \frac{5\pi}{2}$

A1 $t = 15$

**(2)**

**(Total for Question 10 is 9 marks)**

11. a. Find the binomial expansion of $(4-x)^{-\frac{1}{2}}$, up to and including the term in $x^2$.

To use the binomial expansion from the formula book, put into the form $(1+x)^n$.

$$(4-x)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}}$$

$$\frac{1}{2}\left(1-\frac{x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1-\frac{1}{2}\left(-\frac{x}{4}\right)+\frac{-\frac{1}{2}\left(-\frac{3}{2}\right)}{2}\left(\frac{-x}{4}\right)^2+\cdots\right)$$

Simplifying:

$$= \frac{1}{2}\left(1+\frac{x}{8}+\frac{3}{8}\left(\frac{x^2}{16}\right)+\cdots\right)$$

$$\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\cdots$$

B1 For taking out a factor of $4^{-\frac{1}{2}}$ or $\frac{1}{2}$

M1 For the binomial expansion $n = -\frac{1}{2}$ and the term $-\frac{x}{4}$. Condone sign slips

M1 Any (unsimplified) form of the binomial expansion. Ignore the factor before the bracket. The bracketing must be correct but it is acceptable for them to recover from "missing" brackets for full marks.

A1 correct solution only $\frac{1}{2}+\frac{x}{16}+\frac{3x^2}{256}+\cdots$

**(4)**

Given that the binomial expansion of $\mathrm{f}(x) = \sqrt{\frac{1+2x}{4-x}}$ , $|x| < \frac{1}{4}$ , is

$$\frac{1}{2} + \frac{9}{16}x - Ax^2 + \cdots$$

c. Show that the value of the constant $A$ is $\frac{45}{256}$

$$f(x) = (1+2x)^{\frac{1}{2}} \times (4-x)^{-\frac{1}{2}}$$

$$(1+2x)^{\frac{1}{2}} = 1 + \frac{1}{2}(2x) + \frac{\frac{1}{2} \times -\frac{1}{2}}{2}(2x)^2 = 1 + x - \frac{1}{2}x^2$$

$$f(x) = \left(1 + x - \frac{1}{2}x^2\right)\left(\frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256}\right)$$

$$= \frac{1}{2} + \frac{9x}{16} - \frac{45x^2}{256} + \cdots$$

$$A = \frac{45}{256}$$

B1 An attempt for multiplying $(1+2x)^{\frac{1}{2}} \times (4-x)^{-\frac{1}{2}}$

M1 Award for an attempt at the binomial expansion

And for multiplying their two expansions to reach a quadratic.

A1 $A = \frac{45}{256}$

**(3)**

c. By substituting $x = \frac{1}{4}$ into the answer for (b) find an approximate for $\sqrt{10}$, giving your answer to 3 decimal places.

$$f\left(\frac{1}{4}\right) = \sqrt{\frac{1+2(0.25)}{4-0.25}} = \frac{\sqrt{10}}{5}$$

Substituting $x = \frac{1}{4}$ into the binomial expansion gives

$$\frac{\sqrt{10}}{5} \approx \frac{1}{2} + \frac{9(0.25)}{16} - \frac{45(0.25)^2}{256}$$

$$\frac{\sqrt{10}}{5} \approx 0.62964$$

$$\sqrt{10} \approx 3.14819$$

M1 Substitutes $x = \frac{1}{4}$ into both sides and attempts to find at least one side. As the left hand side is $\frac{\sqrt{10}}{5}$ they may multiply by 5 first which is acceptable.

A1 Finds both sides leading to a correct equation $\frac{\sqrt{10}}{5} = \frac{2579}{4096}$

A1 $\sqrt{10} = \frac{12695}{4096} = 3.148193 \approx 3.148$

**(3)**

**(Total for Question 11 is 10 marks)**

12. The table shows the average weekly pay of a footballer at a certain club on 1 August 1990 and 1 August 2010.

| **Year** | 1990 | 2010 |
|---|---|---|
| **Average weekly pay** | £2500 | £50000 |

The average weekly pay of a footballer at this club can be modelled by the equation

$$P = Ak^t$$

where £$P$ is the average weekly pay $t$ years after 1 August 1990, and $A$ and $k$ are constants.

a. i. Write down the value of $A$.

$$2500 = Ak^0$$
$$2500 = A$$

B1 Substitutes $t = 0, P = 2500$ into $P = Ak^t$ to reach a value of $A$.

**(1)**

ii. Show that the value of $k$ is 1.16159, correct to five decimal places.

$$50000 = 2500k^{20}$$
$$20 = k^{20}$$
$$k = 20^{\frac{1}{20}} = 1.161586\ldots$$
$$= 1.16159 \text{ to 5dp}$$

M1 Substitutes $t = 20, A = 2500$ and $P = 50000$ into $P = Ak^t$ to reach a value of $k$.

A1 $k = 1.16159$

**(2)**

b. With reference to the model, interpret

i. the value of the constant $A$,

B1 states that $A$ is the weekly pay of a footballer on 1st August 1990.

The statement must reference the footballer, his weekly pay/wage and '0' time.

ii. the value of the constant $k$,

B1 States that $k$ is the rate at which the weekly pay of the footballer rises each year.

**(2)**

Using the model,

c. find the year in which, on 1 August, the average weekly pay of a footballer at this club will first exceed £100000.

$$100000 = 2500 \times 1.16159^t$$

$$40 = 1.16159^t$$

$$\log 40 = \log 1.16159^t$$

$$\log 40 = \mathrm{t} \log 1.16159$$

$$t = 24.627$$

So the weekly pay of a footballer exceeds £100000 in 2014

M1 Using the model $100000 = 2500 \times 1.16159^t$ and proceeds to $1.16159^t = k$

M1 Correct method to find $t$

A1 $t = 24.627 \ldots$ or $t = \log_{1.16159} 40$

A1 States in the year 2014

**(4)**

**(Total for Question 12 is 9 marks)**

13.

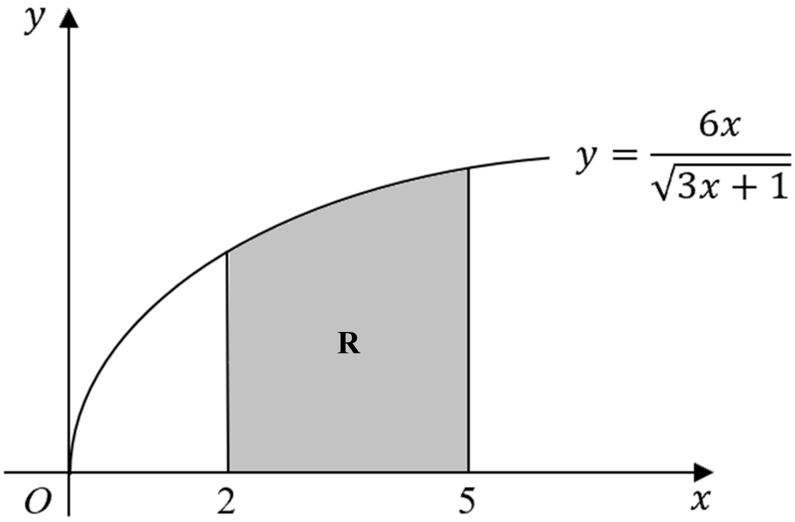



Figure 5 shows a sketch of part of the curve with equation $y = \frac{6x}{\sqrt{3x+1}}$ , $x \geq 0$

The finite region **R**, shown shaded in figure 5 is bounded by the curve, the $x$-axis and the lines $x = 2$ and $x = 5$.

Use the substitution $u = 3x + 1$ to find the exact area of **R**.

$$u = 3x + 1 \Rightarrow \frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$$

$$R = \int_2^5 \frac{6x}{\sqrt{3x+1}} dx$$

$$\int_{x=2}^{x=5} \frac{2u-2}{\sqrt{u}} dx = \int_7^{16} \frac{2u-2}{3\sqrt{u}} du = \int_7^{16} \frac{2}{3} u^{\frac{1}{2}} - \frac{2}{3} u^{-\frac{1}{2}}$$

$$R = \left[\frac{4}{9} u^{\frac{3}{2}} - \frac{4}{3} u^{\frac{1}{2}}\right]_7^{16}$$

$$R = \frac{16}{9}(13 - \sqrt{7})$$

B1 $\frac{\mathrm{d}u}{\mathrm{d}x} = 3$

M1 $\frac{6x}{\sqrt{3x+1}}$ becoming $\frac{2(u-1)}{u^{\frac{1}{2}}}$

A1 $\frac{6x}{\sqrt{3x+1}}\,dx$ becoming $\frac{2(u-1)}{3u^{\frac{1}{2}}}\,\mathrm{d}u$

M1 An attempt to divide the two terms by $u$

$$\frac{2}{3}(u^{\frac{1}{2}} - u^{-\frac{1}{2}})$$

A1 $\int \frac{2}{3}\left(u^{\frac{1}{2}} - u^{-\frac{1}{2}}\right)\,\mathrm{d}u = \frac{2}{3}\left(\frac{u^{\frac{3}{2}}}{\frac{3}{2}} - \frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)$

M1 Some evidence of limits of 16 and 7 in $u$

A1 Exact answer $\frac{16}{9}(13 - \sqrt{7})$

**(Total for Question 13 is 7 marks)**

14. A curve $C$ has parametric equations

$$x = 1 - \cos t, \qquad y = 2\cos 2t, \qquad 0 \le t < \pi$$

a. Show that the cartesian equation of the curve can be written as $y = k(1-x)^2 - 2$ where $k$ is an integer.

Using the trigonometric identity $\cos 2t = 2\cos^2 t - 1$:

$$\frac{y}{2} = 2(1-x)^2 - 1 \Rightarrow y = 4(1-x)^2 - 2$$

M1 Attempts to use $\cos 2t = 2\cos^2 t - 1 \Rightarrow \frac{y}{2} = 2(1-x)^2 - 1$

A1 proceeds to correct answer

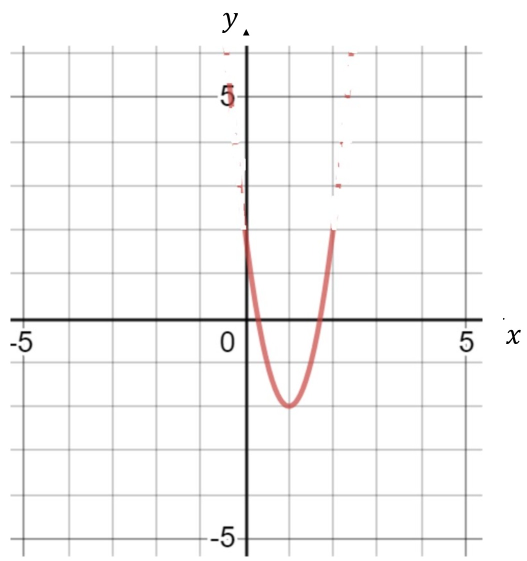
**(2)**

b. i. Sketch the curve C.

ii. Explain briefly why C does not include all points of $y = k(1-x)^2 - 2, \ x \in \mathbb{R}$.

M1 For sketching a ∪ parabola in quadrant one and four with a minimum in quadrant four.

A1 For sketching a ∪ parabola with a minimum in quadrant four and with end points of (0,2) and (5,2)




B1 A refence to the limits on sin or cos with a link to a restriction on $x$ or $y$.

e.g. As $-1 \le \sin t \le 1$ then $0 \le x \le 2$

As $\sin t \le 1$ then $x \le 2$

As $-1 \le \cos 2t \le 1$ then $-2 \le y \le 2$ **(3)**

The line with equation $y = k - x$, where $k$ is a constant, intersects C at two distinct points.

(c) State the range of values of $k$, writing your answer in set notation.

$$k - x = 4(1 - x)^2 - 2$$

$$k - x = 4(x^2 - 2x + 1) - 2$$

$$4x^2 - 7x + 2 - k = 0$$

For intersection at two distinct points, $b^2 - 4ac > 0$

$$49 - 4(4)(2 - k) > 0$$

$$k > \frac{-17}{16}$$

Because of the cut off for the graph, the maximum value of $k$ can be given by substituting $(0,2)$ into the equation:

$$2 = k - 0 \Rightarrow k = 2$$

So $k = \left\{k: -\frac{17}{16} < k \leq 2\right\}$.

B1 Deduces either the correct that the lower value of $k = -\frac{17}{16}$

Or deduces that $k = 2$, substituting (0,2) into $y = k - x$

M1 For an attempt at the upper value for $k$

Finds where $x + y = k$ meets $y = 4(1 - x)^2 - 2$ once by using an appropriate method.

e.g. $k - x = 4(1 - x)^2 - 2$ and proceeds to a 3 term quadratic.

A1 Correct 3 term quadratic $4x^2 - 7x + 2 - k = 0$

M1 Uses the discriminant condition. Accept $b^2 - 4ac \geq 0$ or $b^2 - 4ac = 0$ and leading to a critical value for $k$.

e.g. $49 - 4 \times 4 \times (2 - k) = 0 \Rightarrow k = -\frac{17}{16}$

A1 Range of values for $k = \left\{k: -\frac{17}{16} < k \leq 2\right\}$

**(5)**

**(Total for Question 14 is 10 marks)**