Level 1 / Level 2 GCSE (9 – 1)

MATHEMATICS

Paper 3 (Calculator)

Higher Tier

Time : 1 hour 30 minutes   **Paper : 1 MA1 / 3H**

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may be used.**
- If your calculator does not have a π button, take the value of π to be 3.142 unless the question instructs otherwise.

Information

- The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

Write your answers in the spaces provided.

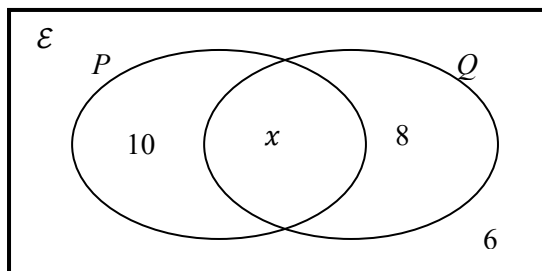**You must write down all the stages in your working.**

1. ε = {students in class 10A}

   P = {students who play piano}

   Q = {students who play guitar}

   There are 30 students in the class.

   The Venn diagram shows number of students.

   a. Find the value of x.

   $$10 + x + 8 + 6 = 30$$

   $$x = 6 \qquad (1)$$

   ………………………………………

   (1)

   b. How many students play piano?

   $$10 + 6 = 16 \qquad (1)$$

   …………………………………..

   (1)

   c. Given that a student plays guitar, work out that this student also plays piano.

   $$P(\text{plays piano}|\text{plays guitar}) = \frac{P(\text{plays piano and plays guitar})}{P(\text{plays guitar})}$$

   $$= \frac{6 \div 30}{14 \div 30}$$

   $$= \frac{3}{7}$$

   ....

   (2)

   (Total for Question 1 is 4 marks)

2. Harry bought a car for £22500.

The car depreciated by 10% each year.

Work out the value of the car 5 years after he bought it.

Give your answer correct to the nearest pound.
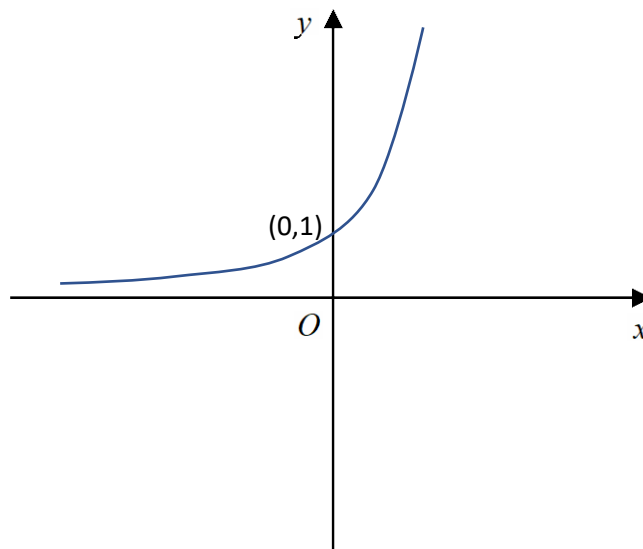
$$22500 \times (\frac{90}{100})^5 = £13286.025$$

(1)

(1)

$$= £13286 \quad (1)$$

…………………………………

(Total for Question 2 is 3 marks)

3. On the grid, sketch the curve with equation $y = 3^x$
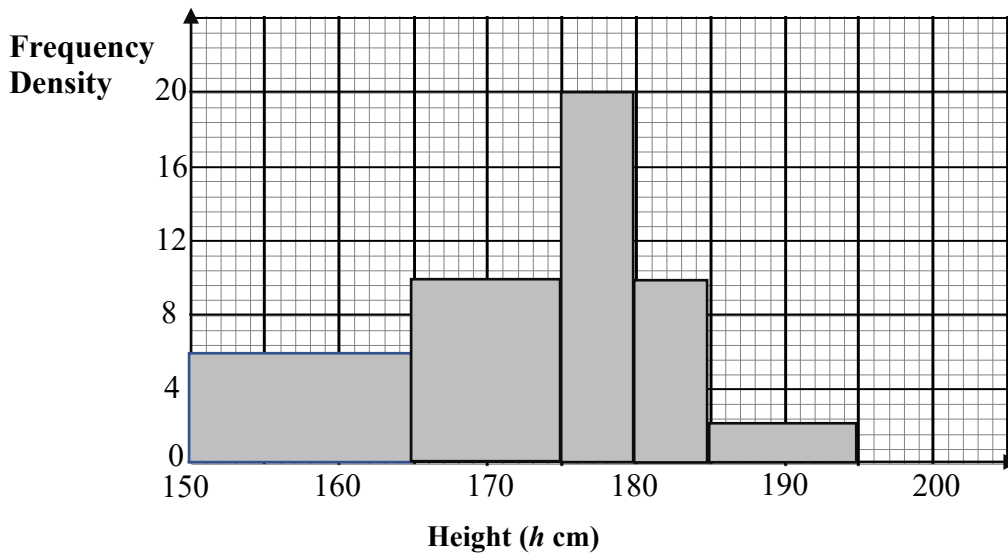Give the coordinates of any points of intersection with the axes.

Shape of the curve (1)   y-intercept   (1)

(Total for Question 3 is 2 marks)

4. The histogram shows the distribution of heights of all the sixth formers in a school.

Height (h cm)

a. Complete the table.

Height (h cm)	Number of students
$150 \leq h < 165$	$6 \times 15 = 90$
$165 \leq h < 175$	$10 \times 10 = 100$
$175 \leq h < 180$	$20 \times 5 = 100$
$180 \leq h < 185$	$10 \times 5 = 50$
$185 \leq h < 195$	$2 \times 10 = 20$

Correct width   (1)

At least 3 correct frequency density × width   (1)

All correct width × frequency density   (1)

(3)

b. How many sixth form students are there in the school?

360   (1)

……………………………………..

(1)

c. Find the class interval that contains the median.

$$\frac{360}{2} = 180$$

The interval class that contains the 180$^{\text{th}}$ student is $165 \leq h < 175$ (1)

(1)

…………………………………

(Total for Question 4 is 5 marks)

5. The table shows the temperatures of several test tubes during a Biology experiment.

Temperatures ($T\ ^0C$)	Frequency
$45 \leq t < 50$	4
$50 \leq t < 55$	8
$55 \leq t < 60$	17
$60 \leq t < 65$	6
$65 \leq t < 70$	2
$70 \leq t < 75$	1

a. Calculate an estimate for the total temperature of the test tubes.

$(47.5 \times 4) + (52.5 \times 8) + (57.5 \times 17) + (62.5 \times 6) + (67.5 \times 2) + (72.5 \times 1) = 2170$

(1)

At least 4 correct mid-points are used in the product   (1)

All product are correct   (1)

……………………………………..

(3)

One of the 38 test tubes is going to be chosen at random.

b. Find the probability that this test tube has a temperature of less than 55 ^{0}C.

$\dfrac{12}{38}$

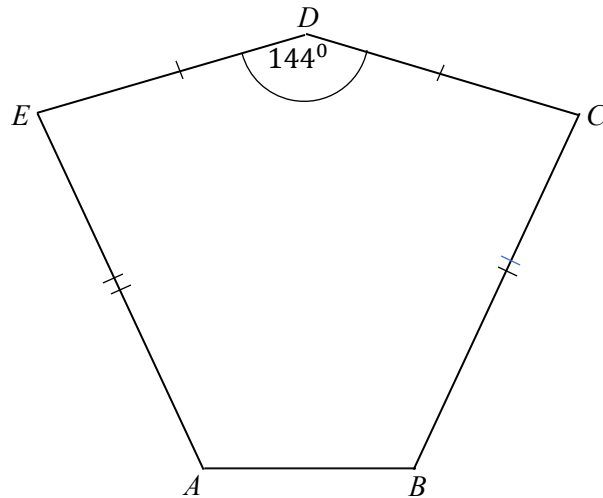Denominator = 38   (1)   fraction (1)

……………………………………..

(2)

(Total for Question 5 is 5 marks)

6. The diagram shows a pentagon.

The diagram has one line of symmetry.

$AE = BC$

$DE = DC$

Angle $EDC = 144^0$

Angle $EAB = 2 \times$ angle AED

Work out the size of angle ABC.

You must show all your working.

Interior angles of a pentagon= 540°

Let angle $AED = x$ angle $EAB = 2x$ (1)

$2x + x + 144 + x + 2x = 540°$ (1)

$x = 66°$ (1)

angle $ABC = 132°$ (1)

(4)

……………………………………

(Total for Question 6 is 4 marks)

7. ABC is a right-angle triangle.
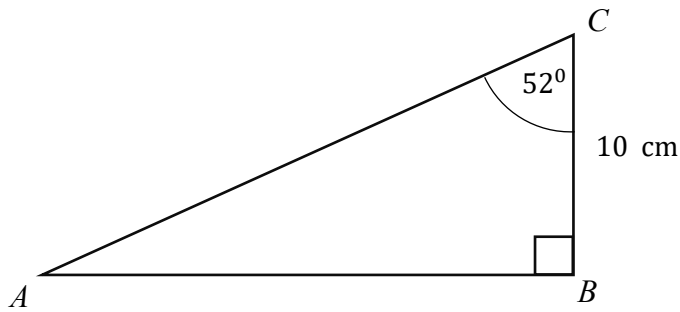
Here is Jamie's method to find the length of AC.

$$\cos 52^0 = \frac{10}{AC}$$

$$AC \cos 52^0 = 10$$

$$AC = \frac{\cos 52^0}{10}$$

$$AC = 0.0616 \text{ cm}$$

a. What mistake has Jamie made in her method?

$AC = \frac{\cos 52^0}{10}$ is incorrect, it should be replaced by $AC = \frac{10}{\cos 52}$ so $AC = 16.2$ cm   (1)

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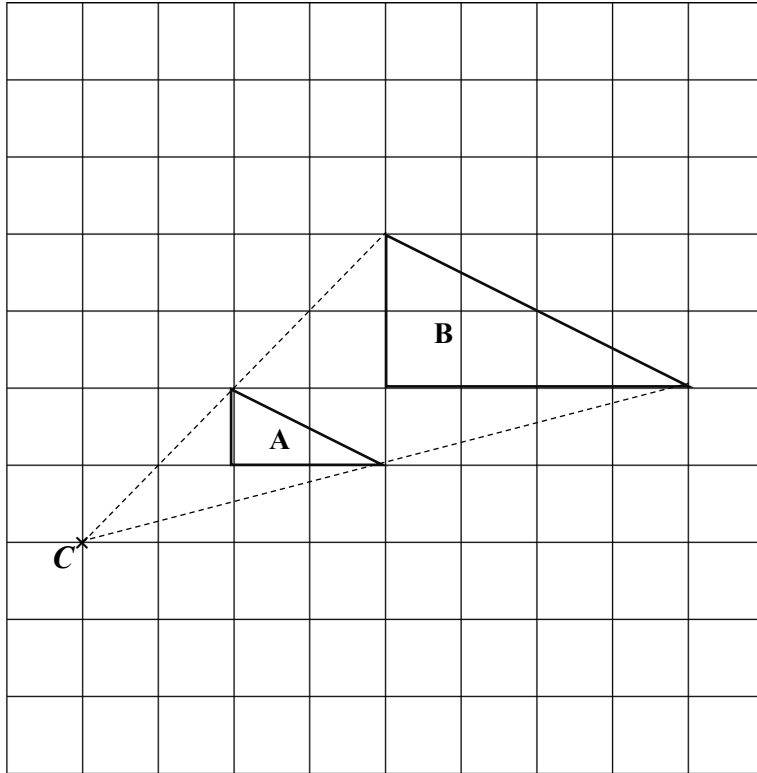
(1)

In the above grid triangle **B** is the image of triangle **A**.

Julie is describing the transformation that maps **A** onto **B**.

This is an enlargement with scale factor $\frac{1}{2}$ with centre **C**.

b. Explain what mistake has she done on describing this transformation?

Scale factor should be 2 instead of $\frac{1}{2}$ (1)

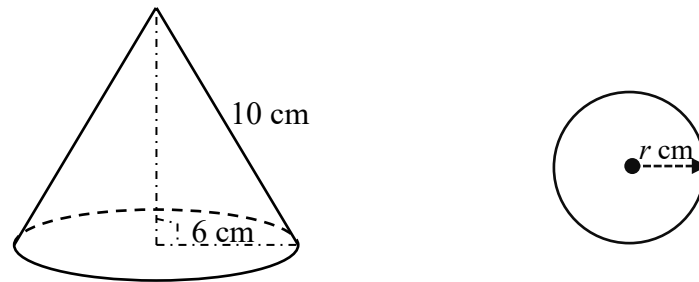…………………………………………………………………………………………...

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(1)

(Total for Question 7 is 2 marks)

8.

The diagram shows two solids.

The base radius of the cone is 6 cm and slant height is 10 cm.

The radius of the sphere is r cm.

a. Show that total surface area of the cone is 96π cm^2.

$$TSA_{cone} = \pi r^2 + \pi r l$$

$$= \pi \times 6^2 + \pi \times 6 \times 10 \qquad (1)$$

$$= 96\pi \qquad (1)$$

……………………………….

(2)

The total surface area of cone is 6 times the total surface area of a sphere.

b. Find the value of r.

$$TSA_{cone} = TSA_{sphere}$$

$$96\pi = 6 \times 4\pi r^2 \qquad (1)$$

$$r^2 = 4 \qquad (1)$$

$$r = 2 \qquad (1)$$

…………………………...

(3)

(Total for Question 8 is 5 marks)

9. The table shows the average house prices, in pounds, of London in each of the years

   2016, 2017 and 2018

Year	2016	2017	2018
Price of House (pounds)	561400	592500	607500

Mira says,

"The percentage increase in the average price of house from 2017 to 2018 is more than the

percentage increase in the average price of house from 2016 to 2017"

a. Is Mira correct?

   You must show how you get your answer.

   Percentage increase from 2016 to 2017: $\frac{592500-561400}{561400} \times 100\% = 5.54\%$ (1)

   Percentage increase from 2017 to 2018: $\frac{607500-592500}{592500} \times 100\% = 2.53\%$ (1)

   $5.54 > 2.53$, so Mira is incorrect (1)

   (1)

                                                                                                (4)

Henri wants to buy a house.

He gets a discount of 5% off the normal price.

Henri pays 570000 pounds for his house after the discount.

b. Work out the discount that Henri gets.

   $x \times \frac{95}{100} = 570000$ (1)

   $x = 570000 \times \frac{100}{95}$

   $x = 600000$ (1)

   $600000 - 570000 = 30000$ (1)

                                                      …………………….pounds

                                                                                                (3)

                                                         (Total for Question 9 is 7 marks)

10. Gerard drove 76 km from Birmingham to Derby.

He then drove from 120 km from Derby to Leeds.

Gerard took 40 minutes from Birmingham to Derby.

Gerard's average speed from Derby to Leeds was 100 km/h.

Work out Gerard's average speed for his total drive from Birmingham to Leeds.

Birmingham to Derby:

Distance: 76 km

Time: 40 min

40 mins $= 0.\dot{6}$ hours   (1)

Derby to Leeds:

Distance: 120km

Speed: 100km/h

$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{120}{100} = 1.2$ hours (1)

Average speed:

$\text{speed} = \frac{\text{distance}}{\text{time}}$

$= \frac{76+120}{1.2+0.\dot{6}} = 105 km/h$ (1)

(1)

…………………………..km/h **(4)**

(Total for Question 10 is 4 marks)

11. Mary wants to work out the value of x in $x^2 = 64$.

Here is Mary's method to find the value of x:

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$x = 8$$

What mistake has Mary made in her method?

x has two values 8 or -8   (1)

……………………………………………………………………………………………..

(1)

(Total for Question 11 is 1 marks)

12. Brass is an alloy made from zinc and copper.

The ratio of the volume of copper to the volume of zinc in the alloy is 5 : 3.

The density of the zinc is 2.5 g/cm³.

The density of copper is 3 g/cm³.

A brass ornament has a volume of 480 cm³.

Calculate the mass of the ornament.

Volume of copper: $\frac{480}{8} \times 5 = 300cm^3$

Mass of copper: mass = density × volume

$= 3 \times 300$

$= 900g$

Volume of zinc: $\frac{480}{8} \times 3 = 180cm^3$

Mass of zinc: $mass$ = density × volume

$= 2.5 \times 180$

$= 450g$

Total mass = 450 + 900 = 1350g   (1)

Both volumes correct (1)
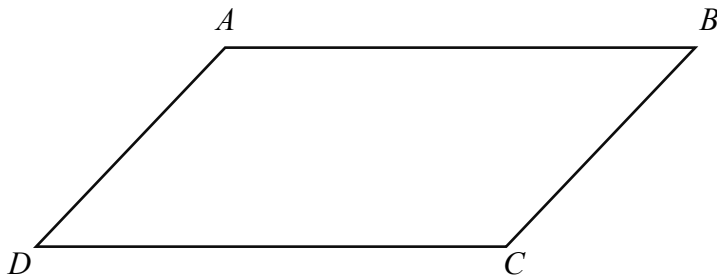
Each mass correct (1), (1)

Correct total mass (1)

…………………………..g

(Total for Question 12 is 4 marks)

13.

In the diagram *AB* is parallel to *CD*.

$AB : AD = \sqrt{2} : 1$ and angle $BAD = 120^0$

Area of the parallelogram is $24\sqrt{6}$ cm^2.

Find the length of *AB* in its simplest surd form.

Let $AD = x$, $AB = \sqrt{2}x$

Area of parallelogram = 2 × Area of triangle *DAB*

$$= 2 \times (\frac{1}{2} \times x \times \sqrt{2}x \times \sin 120)$$

Correct area of triangle formula used (1)

$24\sqrt{6} = \frac{\sqrt{6}}{2}x^2$ (1)

$48\sqrt{6} = \sqrt{6}x^2$

$x^2 = 48$ (1)

$x = 4\sqrt{3}$ (1)

…………………………..cm

(Total for Question 13 is 3 marks)

14. The graph of $y = \cos x$ is translated by the vector $\begin{pmatrix} 60^0 \\ 0 \end{pmatrix}$

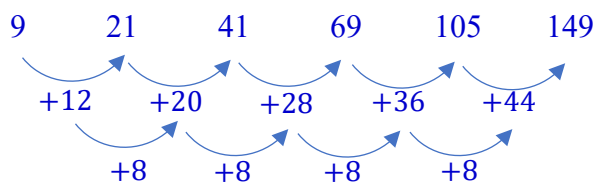Write down the equation of the new graph.

$y = \cos(x \pm 60^0)$ (1)

$y = \cos(x - 60^0)$ (1)   correct answer

…………………………..

(Total for Question 14 is 2 marks)

15. Here are the first six term of a quadratic sequence.

$$9, 21, 41, 69, 105, 149$$

Find an expression, in terms of n, for the nth term of this sequence.

9   21   41   69   105   149

+12   +20   +28   +36   +44

+8   +8   +8   +8

General form for quadratic sequence n^{th} term $an^2 + bn + c$.

$a = \dfrac{2^{nd}\text{difference}}{2} = \dfrac{8}{2} = 4$

1st term of 1st difference $= 3a + b$

$3(4) + b = 12 \Rightarrow b = 0$ (1)

1st term $= a + b + c$

$4 + 0 + c = 9 \Rightarrow c = 5$ (1)

n^{th}term $= 4n^2 + 5$ (1)

…………………………..

(Total for Question 15 is 3 marks)

16. a. Show that $(x + 1)(2x - 3)^2$ can be written in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are integers.

$$(x + 1)(2x - 3)^2 = (x + 1)(4x^2 - 12x + 9) \qquad (1)$$
$$= 4x^3 - 12x^2 + 9x + 4x^2 - 12x + 9 \qquad (1)$$
$$= 4x^3 - 8x^2 - 3x + 9 \qquad (1)$$

(3)

b. Solve $x^2 - \frac{1}{5} < \frac{4}{25}$

$$x^2 < \frac{4}{25} + \frac{1}{5}$$
$$x^2 < \frac{9}{25} \qquad (1)$$

Critical values of x are $\pm \frac{3}{5}$ $\qquad (1)$

$$-\frac{3}{5} < x < \frac{3}{5} \qquad (1)$$

(3)

(Total for Question 16 is 6 marks)

17. $y = x \sin z^0$

$x = 4.2$ correct to 1 decimal place.

$z = 60^0$ correct to 2 significant figures.

a. Calculate the lower bound for value of y.

Write down all the figures on your calculator display.

Minimum value of x: 4.15

Minimum value of z: 59.5

$y = 4.15 \sin 59.5^0$ correct min values used (1)

$\qquad = 3.575761016$ (1)

…………………………..

(2)

b. Give your value of y to an appropriate degree of accuracy.

You must show working to explain how you obtained your answer.

$y = 4.25 \sin 60.5^0$ correct max values used (1)

$= 3.699011708$ (1)

4 correct to 1 significant figure   (1)

…………………………..

(3)

(Total for Question 17 is 5 marks)

18. Find the coordinates of the points at which the line with equation $y = x - 4$ intersects the curve with equation $y^2 = 2x^2 - 17$.

Substitute $y = x - 4$ into $y^2 = 2x^2 - 17$

$(x - 4)^2 = 2x^2 - 17$ (1)

$x^2 - 8x + 16 = 2x^2 - 17$

$x^2 + 8x - 33 = 0$　　(1)

$(x + 11)(x - 3) = 0$　　(1)

$x = -11$ or $x = 3$

Substitute the x values into the equation of the line:
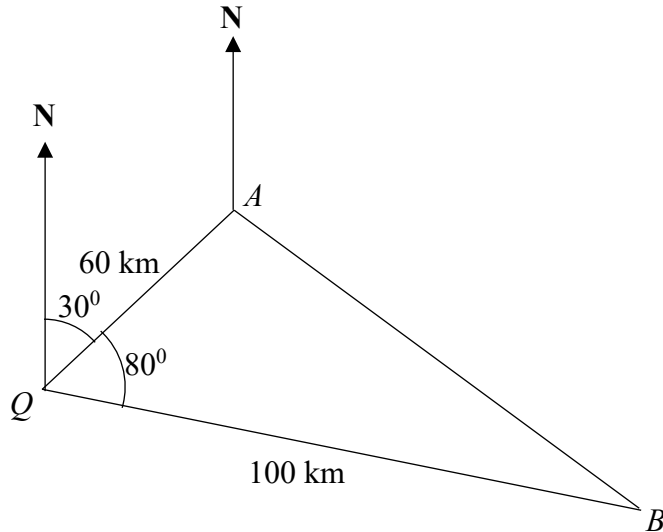
$y = -11 - 4$

$y = -15$ (1)

$y = 3 - 4$

$y = -1$ (1)

……………………………………………………..

(Total for Question 18 is 5 marks)

19.

Two ships leave port Q at the same time. One ship sails 60 km on a bearing of 030^0 to position A. The other ship sails 100 km on a bearing of 110^0 to position B.

a. Calculate the distance AB.

Give your answer correct to 3 significant figures.

Cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$a = AB, b = QA, c = QB,$

$AB^2 = 60^2 + 100^2 - 2 \times 60 \times 100 \times \cos 80^0$ (1)

$AB = 107.3136$

$AB = 107$ to 3 significant figures   (1)

…………………………….(2)

b. Calculate the bearing of B from A.

Give your answer correct to the nearest degree.

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

$a = AB, b = QB$

$\frac{107}{\sin 80} = \frac{100}{\sin QAB}$ (1)

$\sin QAB = 0.92038$

$QAB = 66.98185$ (1)

$NAQ = 180 - 30$ because co-interior angles add to 180° (1)

$NAB = 360 - (150 + 66.98)$ (1)

$NAB = 143.02$ (1)

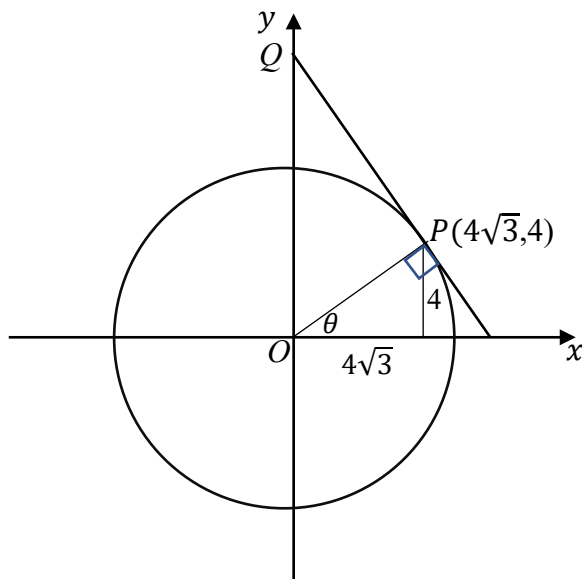$NAB = 143$ to the nearest degree (1)

………………………….

(4)

(Total for Question 19 is 6 marks)

20. The diagram shows a circle, centre O.

PQ is the tangent to the circle at the point P.

P has coordinates $\left(4\sqrt{3}, 4\right)$.

Q has coordinates $(0, y)$.

Find the value of y.

Finding θ:

$\tan \theta = \dfrac{4}{4\sqrt{3}}$

$\theta = 30^0$ (1)

Finding OP:

$OP^2 = 4^2 + \left(4\sqrt{3}\right)^2$

$OP = 8$ (1)

Using trigonometry to find OQ:

$QOP = 60^0$

$\cos 60^0 = \dfrac{8}{y}$ (1)

$y = 16$ (1)

……………………..

(Total for Question 20 is 4 marks)