Write your name here

Surname	Other names
MODEL ANSWERS	

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference **9MA0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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Pearson

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

a)

i) $y = 3x^4 - 8x^3 - 3 \Rightarrow \frac{dy}{dx} = 12x^3 - 24x^2$ (1) (1)

ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$ (1)

b) Stationary point when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 12x^3 - 24x^2 \Rightarrow 12(2)^3 - 24(2)^2 = 12 \times 8 - 24 \times 4 = 0$ (1)

$\Rightarrow$ At $x = 2$, $\frac{dy}{dx} = 0 \Rightarrow x = 2$ is a stationary point. (1)

c) $\frac{d^2y}{dx^2}$ and substitute in $x = 2$, $\frac{d^2y}{dx^2} > 0 \Rightarrow$ Minimum

$\frac{d^2y}{dx^2} < 0 \Rightarrow$ Maximum

$\left.\frac{d^2y}{dx^2}\right|_2 = 36(2)^2 - 48(2) = 144 - 96 = 48$ (1)

$\Rightarrow 48 > 0 \Rightarrow$ Stationary point which is a minimum. (1)

Question 1 continued

(Total for Question 1 is 7 marks)

2.

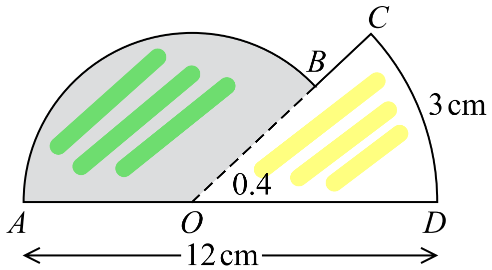

Figure 1

The shape *ABCDOA*, as shown in Figure 1, consists of a sector *COD* of a circle centre *O* joined to a sector *AOB* of a different circle, also centre *O*.

Given that arc length *CD* = 3 cm, $\angle COD = 0.4$ radians and *AOD* is a straight line of length 12 cm,

(a) find the length of *OD*,

(2)

(b) find the area of the shaded sector *AOB*.

(3)

a)

OD = radius

$S = r\theta$ $\quad$ S = arc length = 3cm

r = radius, ?

θ = 0.4 radians

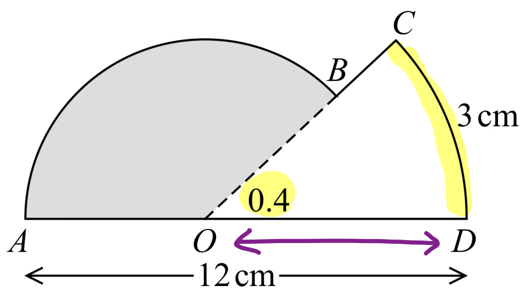

Figure 1

$3 = r \times 0.4$ (1) $\Rightarrow r = 3/0.4 = 7.5\text{cm}$

$\Rightarrow$ OD = 7.5cm (1)

b) Area of Sector : $A = \frac{1}{2}r^2\theta$

$\Rightarrow r = AO = 12 - 7.5 = 4.5\text{cm}$

$\pi = \theta + 0.4 \Rightarrow \theta = \pi - 0.4$ (1)

$\Rightarrow A = \frac{1}{2}(4.5)^2 \times (\pi - 0.4)$ (1)

$\Rightarrow A = 27.8\text{cm}^2$ (1)

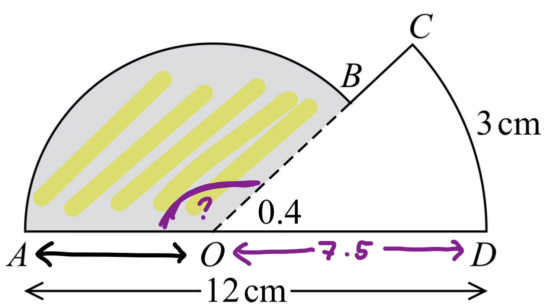

Figure 1

(Total for Question 2 is 5 marks)

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3. A circle C has equation

$$x^2 + y^2 - 4x + 10y = k$$

where k is a constant.

(a) Find the coordinates of the centre of C.
(2)

(b) State the range of possible values for k.
(2)

a) $x^2 + y^2 - 4x + 10y = k$

$\underbrace{x^2 - 4x} + \underbrace{y^2 + 10y} = k$

$(x-2)^2 - 4 + (y+5)^2 - 25 = k$ 1

$x: 2$ and $y: -5$ $\Rightarrow$ $C: \underline{(2, -5)}$ 1

b) What do we know about the radius? $r > 0$

$(x-2)^2 + (y+5)^2 - \cancel{29} = k + 29$ 1 $\Rightarrow$ $k + 29 > 0$

$\Rightarrow k > -29$

$\Rightarrow$ $\underline{k > -29}$ 1

(Total for Question 3 is 4 marks)

4. Given that a is a positive constant and

$$\int_a^{2a} \frac{t+1}{t}\,\mathrm{d}t = \ln 7$$

show that $a = \ln k$, where k is a constant to be found.

(4)

$$\int_a^{2a} \frac{t+1}{t}\,dt = \int_a^{2a} \frac{t}{t} + \frac{1}{t}\,dt = \int_a^{2a} 1 + \frac{1}{t}\,dt = \Big[t + \ln(t)\Big]_a^{2a}$$ 1 1

$$\Rightarrow \big(2a + \ln(2a)\big) - \big(a + \ln(a)\big) = \ln(7)$$ 1

$$\Rightarrow 2a + \ln(2) + \ln(a) - a - \ln(a) = \ln(7)$$

$$\Rightarrow a + \ln(2) = \ln(7)$$

$\log(ab) = \log(a) + \log(b)$

$\log(a/b) = \log(a) - \log(b)$

$$\Rightarrow a = \ln(7) - \ln(2) = \ln(7/2)$$

$$\Rightarrow a = \ln\left(\frac{7}{2}\right) = \ln(k) \quad \text{where } k = \frac{7}{2}.$$ 1

(Total for Question 4 is 4 marks)

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5. A curve C has parametric equations

$$x = 2t - 1, \qquad y = 4t - 7 + \frac{3}{t}, \qquad t \neq 0$$

Show that the Cartesian equation of the curve C can be written in the form

$$y = \frac{2x^2 + ax + b}{x + 1}, \qquad x \neq -1$$

where a and b are integers to be found.

(3)

$C: x = 2t - 1$, $y = 4t - 7 + \frac{3}{t} \Rightarrow y = 4\left(\frac{x+1}{2}\right) - 7 + \frac{3 \times 2}{x+1}$

$x + 1 = 2t$

$\Rightarrow t = \frac{x+1}{2}$

$\Rightarrow y = \frac{4(x+1)}{2} - 7 + \frac{6}{x+1}$ ①

$\Rightarrow y = 2x + 2 - 7 + \frac{6}{x+1}$

$\Rightarrow y = (2x - 5) + \frac{6}{x+1}$

$\Rightarrow y = \frac{(2x-5)(x+1) + 6}{x+1}$ $\quad 2x - 5x = -3x$

$\Rightarrow y = \frac{2x^2 - 3x - 5 + 6}{x+1} = \frac{2x^2 - 3x + 1}{x+1}$ ①

$y = \frac{2x^2 - 3x + 1}{x+1}$, $a = -3$ and $b = 1$ ①

(Total for Question 5 is 3 marks)

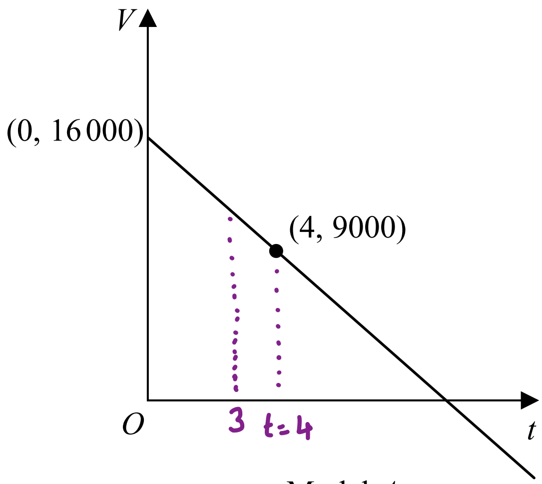
6. A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

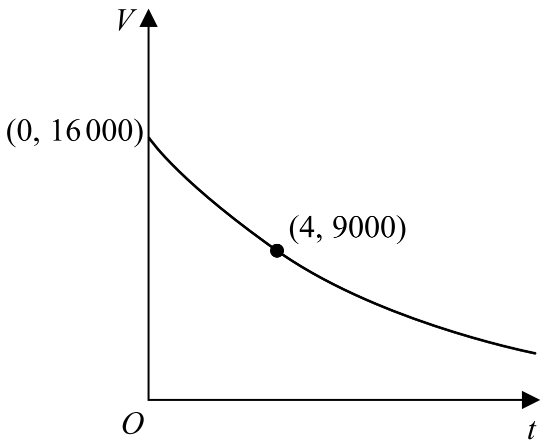
The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



Model A



Model B

(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

a i) $m = \dfrac{9000 - 16000}{4 - 0} = -1750 \Rightarrow y = 16000 - 1750x$

$V = 16000 - 1750t$

$V = 16000 - 1750 \times 3 = \underline{10,750}$ barrels

1

a ii) • $V = 16000 - 1750t$ What happens when $t = 10$?

$V = 16000 - 1750 \times 10 = -1500 \Rightarrow$ This is impossible as $V > 0$. 1

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Question 6 continued

b i) $Y = Ae^{kt} \Rightarrow V = Ae^{kt}$ (1) $e^0 = 1$

$t = 0, V = 16000 \Rightarrow 16000 = Ae^{k\times 0} \Rightarrow \underline{16000} = A$

$t = 4, V = 9000 \Rightarrow 9000 = 16000e^{4k}$ (1) $\frac{9000}{16000} = \frac{9}{16}$

$\Rightarrow e^{4k} = 9/16$

$\Rightarrow 4k = \ln(9/16)$

$\Rightarrow k = \frac{1}{4}\ln(9/16)$ (1)

$\Rightarrow V = \underline{16000e^{1/4\ln(9/16)t}}$ (1)

b ii) $V = 16000e^{\frac{1}{4}\ln(9/16)\times 3}$

$V = 10392.....$

$V = 10,\underline{400}$ barrels (1)

(Total for Question 6 is 7 marks)

7.

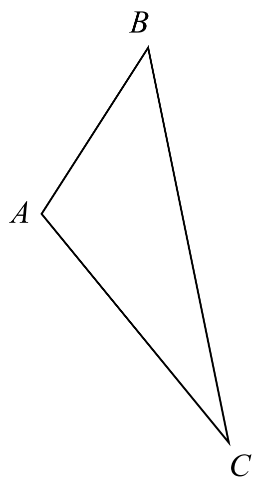


Figure 2

Figure 2 shows a sketch of a triangle ABC.

Given $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{BC} = \mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$,

show that $\angle BAC = 105.9°$ to one decimal place.

(5)

$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}2\\3\\1\end{pmatrix} + \begin{pmatrix}1\\-9\\3\end{pmatrix} = \begin{pmatrix}3\\-6\\4\end{pmatrix}$ ①

$\cos\theta = \dfrac{a \cdot b}{|a||b|}$ let $a = \begin{pmatrix}2\\3\\1\end{pmatrix}$ $b = \begin{pmatrix}3\\-6\\4\end{pmatrix}$

$|a| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$, $|b| = \sqrt{3^2 + (-6)^2 + 4^2} = \sqrt{61}$ ①

$a \cdot b = (2 \times 3) + (3 \times -6) + (1 \times 4) = 6 - 18 + 4 = -8$ ①

$\Rightarrow \cos\theta = \dfrac{-8}{\sqrt{14} \times \sqrt{61}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-8}{\sqrt{14} \times \sqrt{61}}\right)$ ①

$\Rightarrow \theta = 105.887\ldots$

$\theta = 105.9°$ ①

$\Rightarrow \angle BAC = 105.9°$ as required.

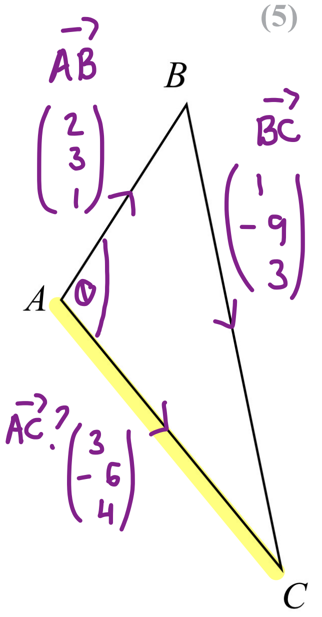


Figure 2

Question 7 continued

(Total for Question 7 is 5 marks)

8. $$f(x) = \ln(2x - 5) + 2x^2 - 30, \quad x > 2.5$$

(a) Show that $f(x) = 0$ has a root α in the interval [3.5, 4]

(2)

A student takes 4 as the first approximation to α.

Given $f(4) = 3.099$ and $f'(4) = 16.67$ to 4 significant figures,

(b) apply the Newton-Raphson procedure once to obtain a second approximation for α, giving your answer to 3 significant figures.

(2)

(c) Show that α is the only root of $f(x) = 0$

(2)

a) $f(x) = \ln(2x-5) + 2x^2 - 30$

$f(3.5) = \ln(2\times3.5-5) + 2(3.5)^2 - 30 = -4.81$

$f(4) = 3.10$ (1) $\Rightarrow$ In the interval $[3.5, 4]$ we see a change in sign $\Rightarrow$ there is a root, α, in this interval. (1)

b) Newton - Raphson : $\boxed{x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}}$

$x_0 = 4$

$f(4) = 3.099$

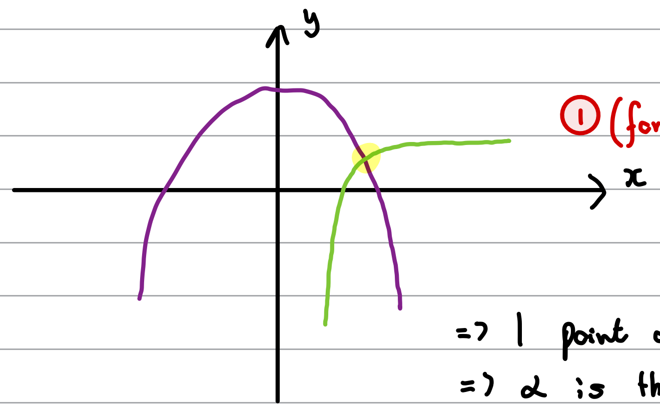
$f'(4) = 16.67$

$\Rightarrow x_1 = 4 - \dfrac{3.099}{16.67}$ (1) $= 3.81409\ldots \Rightarrow x_1 = 3.81$ (1)

c) $f(x) = 0 \Rightarrow \ln(2x-5) + 2x^2 - 30 = 0$

$\Rightarrow \ln(2x-5) = 30 - 2x^2$

$30 - 2x^2$ 



(1) (for sketch)

$\Rightarrow$ 1 point of intersection

$\Rightarrow \alpha$ is the only root. (1)

Question 8 continued

(Total for Question 8 is 6 marks)

9. (a) Prove that

$$\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta, \qquad \theta \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

does not have any real solutions.

(1)

a) $\tan\theta + \cot\theta \equiv 2\operatorname{cosec}2\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

$\equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$ 1 $\equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ 1

$\sin^2\theta + \cos^2\theta = 1$

$\equiv \dfrac{1}{\frac{1}{2}\sin 2\theta}$ 1 $\equiv \dfrac{2}{\sin 2\theta}$

$\frac{1}{2}\sin(2\theta) = \frac{\cancel{2}}{\cancel{2}}\sin\theta\cos\theta$

$\operatorname{Cosec} 2\theta \equiv \dfrac{1}{\sin 2\theta}$

$\equiv 2\operatorname{Cosec} 2\theta$

$\Rightarrow \tan\theta + \cot\theta \equiv 2\operatorname{Cosec} 2\theta$ as required. 1

(b) Hence explain why the equation

$$\tan\theta + \cot\theta = 1$$

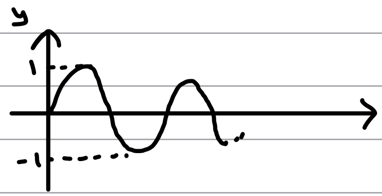
does not have any real solutions.

(1)

$\tan\theta + \cot\theta \equiv 2\operatorname{Cosec}(2\theta) \equiv \dfrac{2}{\sin 2\theta} = 1$

$\Rightarrow \sin 2\theta = 2$

$2\theta = \sin^{-1}(2)$



$-1 \leq \sin 2\theta \leq 1$ 1

$2 > 1$

$\Rightarrow$ No Solution exists.

(Total for Question 9 is 5 marks)

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10. Given that θ is measured in radians, prove, from first principles, that the derivative of $\sin\theta$ is $\cos\theta$

You may assume the formula for $\sin(A \pm B)$ and that as $h \to 0$, $\frac{\sin h}{h} \to 1$ and $\frac{\cos h - 1}{h} \to 0$

(5)

Definition of the derivative : $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$ 1

$\Rightarrow f(\theta) = \sin\theta \quad , \quad f(\theta + h) = \sin(\theta + h)$

Compound Angle Identity : $\sin(\theta + h) = \sin\theta\cos(h) + \sin(h)\cos\theta$ 1

$\Rightarrow f'(\theta) = \lim_{h \to 0} \frac{\sin\theta\cos(h) + \sin(h)\cos\theta - \sin\theta}{h}$ 1

$\Rightarrow f'(\theta) = \lim_{h \to 0} \sin\theta \frac{(\cos(h) - 1)}{h} + \frac{\sin(h)}{h} \cdot \cos\theta$ 1

$\frac{\cos(h) - 1}{h} \to 0$

$\frac{\sin(h)}{h} \to 1$

$\Rightarrow f'(\theta) = \sin\theta \cdot 0 + 1 \cdot \cos\theta = \cos\theta$

$\Rightarrow \underline{f'(\theta) = \cos\theta}$ as required. 1

(Total for Question 10 is 5 marks)

11. An archer shoots an arrow.

The height, H metres, of the arrow above the ground is modelled by the formula

$$H = 1.8 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

where d is the horizontal distance of the arrow from the archer, measured in metres.

Given that the arrow travels in a vertical plane until it hits the ground,

(a) find the horizontal distance travelled by the arrow, as given by this model.
(3)

(b) With reference to the model, interpret the significance of the constant 1.8 in the formula.
(1)

(c) Write $1.8 + 0.4d - 0.002d^2$ in the form

$$A - B(d - C)^2$$

where A, B and C are constants to be found.
(3)

It is decided that the model should be adapted for a different archer.

The adapted formula for this archer is

$$H = 2.1 + 0.4d - 0.002d^2, \qquad d \geqslant 0$$

Hence or otherwise, find, for the adapted model

(d) (i) the maximum height of the arrow above the ground.

(ii) the horizontal distance, from the archer, of the arrow when it is at its maximum height.
(2)

a) $H = 1.8 + 0.4d - 0.002d^2$

$H = 0 \Rightarrow \quad -0.002d^2 + 0.4d + 1.8 = 0$ ①

Quadratic Formula : $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad a = -0.002, \; b = 0.4, \; c = 1.8$

$\Rightarrow \dfrac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(1.8)}}{2(-0.002)}$ ① $\Rightarrow$

+ve: $d = -4.403 \Rightarrow$ Not Valid Since $d < 0$.

-'ve: $d = 204.403 \Rightarrow d = \underline{\underline{204\,m}}$ ①

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Question 11 continued

b)

$H = 1.8 + 0.4d - 0.002d^2.$

$d = 0 \Rightarrow H = 1.8 \Rightarrow 1.8$ is the initial height of the arrow above the ground. ①

c) $1.8 + 0.4d - 0.002d^2$

$\Rightarrow -0.002(d^2 - 200d) + 1.8$ ①   * $(d-100)^2 - 10{,}000$

$\Rightarrow -0.002\left[(d-100)^2 - 10{,}000\right] + 1.8$ ①

$\Rightarrow -0.002(d-100)^2 + 20 + 1.8 \Rightarrow$ $A = 21.8$

$\Rightarrow -0.002(d-100)^2 + 21.8$ ① $B = 0.002$

$C = 100$

d i) $H = 2.1 + 0.4d - 0.002d^2$

$\Rightarrow$ Y-Coordinate of the turning $= 2.1 + 20$

$\Rightarrow$ Max Height $= \underline{22.1\text{m}}$ ①

previously $H = 1.8 + 0.4d - 0.002d^2$

and $H = -0.002(d-100)^2 + 21.8$

$\Rightarrow$ Turning Point

$(100, 21.8)$

$1.8 + 20$

d ii) $2.1 + 0.4d - 0.002d^2 = 22.1$

$\Rightarrow -0.002d^2 + 0.4d - 20 = 0$

$\Rightarrow \dfrac{-0.4 \pm \sqrt{(0.4)^2 - 4(-0.002)(-20)}}{2(-0.002)} \Rightarrow$ +ve: $d = 100$ −ve: $d = 100$

$\Rightarrow \underline{d = 100\text{m}}$ ①

(Total for Question 11 is 9 marks)

12. In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \qquad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10} N = m \log_{10} T + c$$

giving m and c in terms of the constants a and/or b.

(2)

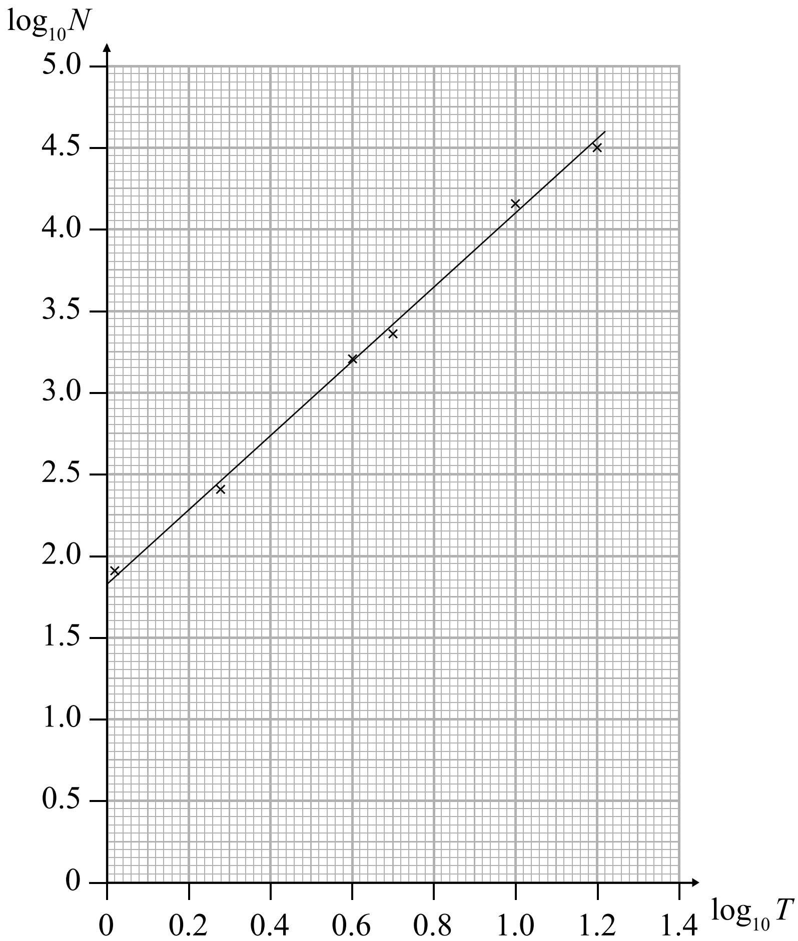

Figure 3

Figure 3 shows the line of best fit for values of $\log_{10} N$ plotted against values of $\log_{10} T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

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a) $N = aT^b$

$\log(ab) = \log(a) + \log(b)$

$\log(a^m) = m\log(a)$

$\log_{10}(N) = \log_{10}(aT^b)$

$= \log_{10}(a) + \log_{10}(T^b)$ (1)

$= \log_{10}(a) + b\log_{10}(T)$

$\Rightarrow \log_{10}(N) = m\log_{10}(T) + c$ where $m = b$ and $c = \log_{10}(a)$ (1)

b) $\log_{10} N = m\log_{10} T + c$

$N = aT^b$ $T = 3$

C : y-intercept $\Rightarrow C = 1.7$

$C = \log_{10}(a) \Rightarrow a = 10^{1.7} \approx 50.12$ $\Rightarrow a = 50.12$ (1)

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{4.5 - 1.8}{1.2 - 0.1} = 2.46$ (1)

$\Rightarrow b = 2.46$

$\Rightarrow N = 50.12(3)^{2.46}$ (1) $= 747.7...$

$= 750$ microbes (1)

c) $N = 1000000 \Rightarrow \log_{10}(N) = \log_{10}(1000000) = 6$ (1)

$6 > 5.0$, which is outwith the data shown on the graph, which means that we can't extrapolate the data/graph, meaning that we can't assume that the model still holds. (1)

d) $N = aT^b$

let $T = 1 \Rightarrow N = a\,1^b$

$\Rightarrow N = a$

$\Rightarrow$ a is the number of microbes one day after the start of the experiment. (1)

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13. The curve C has parametric equations

$$x = 2\cos t, \quad y = \sqrt{3}\cos 2t, \quad 0 \leqslant t \leqslant \pi$$

(a) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t.

(2)

The point P lies on C where $t = \frac{2\pi}{3}$

The line l is the normal to C at P.

(b) Show that an equation for l is

$$2x - 2\sqrt{3}y - 1 = 0$$

(5)

The line l intersects the curve C again at the point Q.

(c) Find the exact coordinates of Q.

You must show clearly how you obtained your answers.

(6)

a) $x = 2\cos t$ and $y = \sqrt{3}\cos(2t)$ $\qquad \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ 1

$\frac{dx}{dt} = -2\sin t \qquad \frac{dy}{dt} = 2 \times \sqrt{3} \times -\sin(2t)$

$\frac{dy}{dt} = -2\sqrt{3}\sin(2t) \qquad \sin(2t) = 2\sin t\cos t$

$= \frac{dy}{dx} = \frac{-2\sqrt{3}\sin(2t)}{-2\sin(t)} = \frac{\sqrt{3}(2\sin t\cos t)}{\sin(t)} = \frac{2\sqrt{3}\cancel{\sin t}\cos t}{\cancel{\sin} t}$

$\Rightarrow \frac{dy}{dx} = \underline{2\sqrt{3}\cos(t)}$ 1

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Question 13 continued

b) $\frac{dy}{dx} = 2\sqrt{3}\cos(t) = 2\sqrt{3}\cos\left(\frac{2\pi}{3}\right) = -\sqrt{3}$ (1)

Gradient of the normal $= -\frac{1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = m$ (1)

$t = \frac{2\pi}{3}$ and $x = 2\cos t$ and $y = \sqrt{3}\cos(2t)$

$\Rightarrow x = 2\cos\left(\frac{2\pi}{3}\right)$ $\quad y = \sqrt{3}\cos\left(2\times\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\Rightarrow x = -1$ (1)

$y - \left(-\frac{\sqrt{3}}{2}\right) = \frac{1}{\sqrt{3}}\left(x-(-1)\right) \Rightarrow y = \frac{1}{\sqrt{3}}x + \frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{2}$ (1)

$\Rightarrow y = \frac{1}{\sqrt{3}}x - \frac{\sqrt{3}}{6}$

$\times\sqrt{3}$ $\Rightarrow \sqrt{3}y = x - \frac{1}{2}$

$\times 2$ $\Rightarrow 2\sqrt{3}y = 2x - 1$

$\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ as required. (1)

c) $x = 2\cos t$ $\quad y = \sqrt{3}\cos(2t)$

Eq of line l: $2x - 2\sqrt{3}y - 1 = 0$

$\Rightarrow 2(2\cos t) - 2\sqrt{3}(\sqrt{3}\cos(2t)) - 1 = 0$ (1) $\quad 6\cos(2t) = 6(2\cos^2 t - 1)$

$\Rightarrow 4\cos t - 6\cos(2t) - 1 = 0$ $\quad = 12\cos^2 t - 6$

$\Rightarrow 4\cos t - 12\cos^2 t + 6 - 1 = 0$ (1)

$\times -1$ $\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$ (1)

Now, let $\theta = \cos t$

$12\theta^2 - 4\theta - 5 = 0 \Rightarrow \theta = \frac{4 \pm \sqrt{(-4)^2 - 4(12)(-5)}}{2\times 24}$

+ve $\sqrt{\ }$: $\theta = \frac{5}{6}$, -ve $\sqrt{\ }$: $\theta = -\frac{1}{2}$

$\cos t = \frac{5}{6}$ $\quad \cos t = -\frac{1}{2} \Rightarrow t = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow$ ignore solution. (1)

$x = 2\cos t$ and $y = \sqrt{3}\cos(2t)$ $\quad 2t = \cos^{-1}(5/6) \times 2$

$x = 2\times\frac{5}{6} = \frac{5}{3}$ $\quad y = \sqrt{3}\cos\left(\cos^{-1}(5/6)\times 2\right) = \frac{7\sqrt{3}}{18}$ (1)

$Q : \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$ (1)

Question 13 continued

Question 13 continued

(Total for Question 13 is 13 marks)

14.

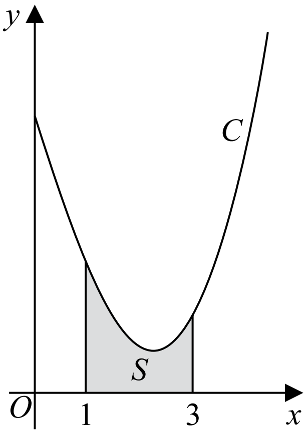

Figure 4

Figure 4 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 \ln x}{3} - 2x + 5, \quad x > 0$$

The finite region S, shown shaded in Figure 4, is bounded by the curve C, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$

The table below shows corresponding values of x and y with the values of y given to 4 decimal places as appropriate.

	1/2	1/2	1/2	1/2	
x	1	1.5	2	2.5	3
y	3	2.3041	1.9242	1.9089	2.2958
	y_0	y_1	y_2	y_3	y_4

(a) Use the trapezium rule, with all the values of y in the table, to obtain an estimate for the area of S, giving your answer to 3 decimal places.

(3)

(b) Explain how the trapezium rule could be used to obtain a more accurate estimate for the area of S.

(1)

(c) Show that the exact area of S can be written in the form $\frac{a}{b} + \ln c$, where a, b and c are integers to be found.

(6)

(In part c, solutions based entirely on graphical or numerical methods are not acceptable.)

a) $A = \frac{1}{2} \times h\left[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$ Trapezium Rule

$h = 1.5 - 1 = 2 - 1.5 = 0.5 \Rightarrow h = 0.5$ 1

$A = \frac{1}{2} \times \frac{1}{2}\left[(3 + 2.2958) + 2(2.3041 + 1.9242 + 1.9089)\right] = 4.39255$ 1

$\Rightarrow$ Area of S is $\underline{4.393}$ (3 d.p) 1

Question 14 continued

b) • h is the width of intervals

⇒ Option 1: decrease h (width of the strips) (1)
Option 2: increase the number of strips

c)
$y = \frac{x^2 \ln x}{3} - 2x + 5$

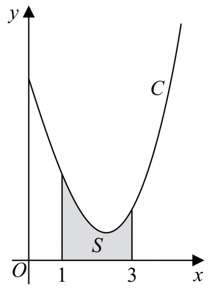

Figure 4

$A = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \, dx$ Integration by parts: (1)

$\int \frac{x^2 \ln x}{3} \, dx = f(x) \cdot g(x) - \int f(x) \cdot g'(x) \, dx$

let $f'(x) = \frac{x^2}{3}$ $f(x) = \frac{x^3}{9}$ $\Rightarrow \int \frac{x^2 \ln x}{3} \, dx = \frac{x^3}{9} \ln(x) - \int \frac{x^3}{9} \cdot \frac{1}{x} \, dx$ (1)

$g(x) = \ln(x)$ $g'(x) = \frac{1}{x}$ $= \frac{x^3}{9} \ln(x) - \frac{1}{9} \int x^2 \, dx$

$= \frac{x^3}{9} \ln(x) - \frac{x^3}{27} + C$ (1)

$\Rightarrow A = \int_1^3 \frac{x^2 \ln x}{3} - 2x + 5 \, dx$

$\Rightarrow A = \left[\frac{x^3}{9} \ln x - \frac{x^3}{27} - x^2 + 5x \right]_1^3$ (1) $= \left[\frac{3^3}{9} \ln(3) - \frac{3^3}{27} - 9 + 15 \right] - \left[\underbrace{\frac{1}{9} \ln(1)}_{=0} - \underbrace{\frac{1}{27} - 1 + 5}_{} \right]$

$-1 - 9 + 15$ $\ln(1) = 0$ $\frac{107}{27}$

$\Rightarrow A = (3\ln(3) + 5) - (107/27)$

$*\; a \ln(b) = \ln(b^a)$

$\Rightarrow A = 3\ln(3) + \frac{28}{27}$

$\Rightarrow A = \ln(27) + \frac{28}{27}$ (1) $a = 28$, $b = 27$ and $c = 27$. (1)

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15.

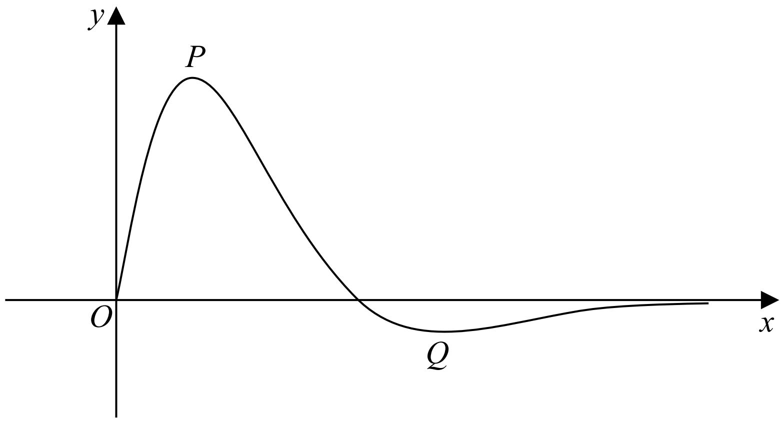

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{4\sin 2x}{e^{\sqrt{2}x-1}}, \quad 0 \leqslant x \leqslant \pi$$

The curve has a maximum turning point at P and a minimum turning point at Q as shown in Figure 5.

(a) Show that the x coordinates of point P and point Q are solutions of the equation

$$\tan 2x = \sqrt{2}$$

(4)

(b) Using your answer to part (a), find the x-coordinate of the minimum turning point on the curve with equation

(i) $y = \text{f}(2x)$.

(ii) $y = 3 - 2\text{f}(x)$.

(4)

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Question 15 continued

a) $f(x) = \dfrac{4\sin 2x}{e^{\sqrt{2}x-1}}$

Quotient Rule : 1

$f'(x) = \dfrac{h'(x)\cdot g(x) - h(x)\cdot g'(x)}{(g(x))^2}$

• Stationary Point when $\boxed{f'(x) = 0}$

let $h(x) = 4\sin 2x$ $\quad h'(x) = 8\cos 2x$

$g(x) = e^{\sqrt{2}x-1}$ $\quad g'(x) = \sqrt{2}e^{\sqrt{2}x-1}$

$\Rightarrow f'(x) = \dfrac{8\cos 2x \cdot e^{\sqrt{2}x-1} - 4\sin 2x\cdot\sqrt{2}e^{\sqrt{2}x-1}}{(e^{\sqrt{2}x-1})^2} = 0$ 1

$\Rightarrow 8\cos 2x \cdot e^{\sqrt{2}x-1} - 4\sqrt{2}\sin 2x e^{\sqrt{2}x-1} = 0$

$\Rightarrow e^{\sqrt{2}x-1}(8\cos 2x - 4\sqrt{2}\sin 2x) = 0$ 1

$\Rightarrow 8\cos 2x - 4\sqrt{2}\sin 2x = 0$

$\Rightarrow 8\cos 2x = 4\sqrt{2}\sin 2x$

$\Rightarrow 8 = \dfrac{4\sqrt{2}\sin 2x}{\cos 2x} \Rightarrow \dfrac{\sin 2x}{\cos 2x} = \dfrac{8}{4\sqrt{2}}$

* $\tan 2x = \dfrac{\sin 2x}{\cos 2x}$

$\Rightarrow \underline{\underline{\tan(2x) = \sqrt{2}}}$ as required 1

b) i) $y = f(2x)$

For $y = f(x) \Rightarrow \tan 2x = \sqrt{2}$

For $y = f(2x) \Rightarrow \tan 4x = \sqrt{2}$

$x = \dfrac{\tan^{-1}\sqrt{2}}{4} + \dfrac{\pi}{4}$ 1

$\underline{\underline{x = 1.024}}$ 1

ii) $y = 3 - 2f(x) \Rightarrow \tan 2x = \sqrt{2}$

$x = \dfrac{\tan^{-1}(\sqrt{2})}{2}$ 1 $= \underline{\underline{0.478}}$ 1

Question 15 continued

(Total for Question 15 is 8 marks)

TOTAL FOR PAPER IS 100 MARKS