M1.(a) (i) Distance travelled in muons' frame of reference

$= 10700(1-0.996^2)^{1/2} = 956$ m ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Time in Earth frame of reference

$= 10700 / (0.996 \times 3 \times 10^8) = 3.581 \times 10^{-5}$ s ✓

Time taken in muons' frame of reference = 3.2 μs ✓

This is 2 half-lives so number reaching Earth = 250 ✓

OR

Half-life in Earth frame of reference

$= 1.6 \times 10^{-6} / (1-0.996^2)^{1/2} = 17.9 \times 10^{-6}$ s ✓

Time taken = 35.8×10^{-6} s ✓

This is 2 half lives so number reaching Earth = 250 ✓

OR

Distance travelled in muons' frame of reference

$= 10700(1-0.996^2)^{1/2} = 956$ m ✓

Distance the muon travels in one half-life in muons reference frame

$= 0.996 \times 3 \times 10^8 \times 1.6 \times 10^{-6} = 478$ m ✓

Therefore 2 half-lives elapse to travel 956 m so number = 250 ✓

OR

Decay constant in muon frame of reference

Or decay constant in the Earth frame of reference ✓

Uses the corresponding elapsed time and decay constant in

$N = N_0 e^{-\lambda t}$ ✓

Arrives at 250 ✓

All steps in the working must be seen

Award marks according to which route they appear to be taking

The number left must be deduced from the correct time that has elapsed in the frame of reference they are using

3

(ii)

	✓ if correct
For an observer in a laboratory on Earth the distance travelled by a muon is greater than the distance travelled by the muon in its frame of reference	✓
For an observer in a laboratory on Earth time passes more slowly than for a muon in its frame of	

reference	
For an observer in a laboratory on Earth, the probability of a muon decaying each second is lower than it is for a muon in its frame of reference	

1

(b) (i) Total energy = $9.11 \times 10^{-31} \times (3 \times 10^8)^2 / (1-0.98^2)^{1/2}$ ✓
4.12×10^{-13} J seen to 2 or more sf ✓
Show that so working must be seen

2

(ii) Change = 7.5×10^{-14} J
V = 469 (470) kV allow ecf using their answer to (i) ✓
ecf is their ((i) −3.37) × 10^{-13}) / 1.6 × 10^{-19}
Using 4 × 10^{-13} gives 394 (390) kV
Using 3.9 × 10^{-13} gives 331(330) kV
Do not allow 1 sf answer

1

[7]

M2.(a) speed of light in free space independent of motion of source and / or the observer ✓
and of motion of observer

1

(b) laws of physics have the same form in all inertial frames
laws of physics unchanged from one inertial frame to another ✓

1

(c) time taken($= \frac{\text{distance}}{\text{speed}} = \frac{34\text{ m}}{0.95 \times 3.0 \times 10^8\text{ m s}^{-1}}$)$=1.2 \times 10^{-7}$ s ✓

1

(d) $t = \frac{18\text{ ns}}{(1 - 0.95^2 c^2 / c^2)^{1/2}}$ ✓
Allow substitution for this mark

1

time taken for π meson to pass from one detector to the other = 58 ns ✓ 1

2 half-lives (approximately) in the detectors' frame of reference. ✓ 1

two half-lives corresponds to a reduction to 25 % so 75% of the π mesons passing the first detector do not reach the second detector. ✓
OR
Appreciation that in the lab frame of reference the time is about 6 half-lives had passed✓ 1

In 6 half-lives 1 / 64 left so about 90% should have decayed✓

Clear conclusion made
Either Using special relativity gives agreement with experiment
or Failure to use relativity gives too many decaying (WTTE) 1

[8]

M3. (a) c is the same, regardless of the speed of the light source or the observer **(1)** 1

(b) distance between detectors in rest frame of particles
$(= 25 \times (1 - 0.98^2)^{1/2}) = 5.0$ m **(1)**

time taken in rest frame of particles $\left(= \frac{distance}{speed} = \frac{5.0}{0.98c}\right) = 1.7 \times 10^{-8}$ s **(1)**

time taken to decrease by ¼ = 2 half lives **(1)**

half life $(= 1.7 \times 10^{-8}/2) = 8.5 \times 10^{-9}$ s **(1)**

[**alternatively**

time taken in rest frame of detectors $\left(= \frac{distance}{speed} = \frac{25.0}{0.98c}\right) = 8.5 \times 10^{-8}$ s

time taken in rest frame of particles
(= $8.5 \times 10^{-8} \times (1 - 0.98^2)^{1/2}$) $= 1.7 \times 10^{-8}$ s)]

4

[5]

M4. (i) time taken $\left(\frac{distance}{speed} = \frac{34}{0.95 \times 3.0 \times 10^8}\right) = 1.1(9) \times 10^7$ s **(1)**

(ii) use of $t = \frac{t_0}{(1 - v^2/c^2)^{1/2}}$ where t_0 = 18 ns

and t is the half-life in the detectors' frame of reference **(1)**

$\therefore t = \frac{18 \times 10^{-9}}{(1 - 0.95^2)^{1/2}} = 57(.6) \times 10^{-9}$ s (1)

time taken for π meson to pass from one detector to the other
= 2 half-lives (approx) (in the detectors' frame of reference) **(1)**
2 half-lives correspond to a reduction to 25%,
so 75% of the π mesons passing the first detector
do not reach the second detector **(1)**

alternatives for first 3 marks in (ii)

1. use of $t = \frac{t_0}{\sqrt{(1 - v^2/c^2}}$, where t_o = 18 ns

$= \frac{18}{(1 - 0.95^2)^{1/2}}$ = 57.6(ns)
journey time in detector frame (= *2t)* = *2* × 57.6ns (≈ 2 half-lives)

2. use of t = $\frac{t_0}{\sqrt{(1 - v^2/c^2}}$ where t = 119 ns

= journey time in detector frame

$t_0 = 119\sqrt{1-0.95^2}$ =37ns
journey time in rest frame = 2 × 18 ns (2 half-lives)

[5]

M5. (a) (i) t_0 = 800 (s) **(1)**

(use of $t = t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ gives) $t = 800(1 - 0.994^2)^{-1/2}$**(1)**

= 7300 s **(1)**

(ii) distance (= 0.994*ct* = 0.994 × 3 × 10^8 × 7300)
= 2.2 × 10^{12}m **(1)** (2.18 × 10^{12}m)
(allow C.E. for value of *t* from (i))

4

(b) space twin's travel time = proper time (or t_0) **(1)**

time on Earth, $t = t_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ **(1)**
$t > t_0$
[or time for traveller slows down compared with Earth twin] **(1)**
space twin ages less than Earth twin **(1)**
travelling in non-inertial frame of reference **(1)**

max 3

[7]