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Surname **MODEL SOLUTIONS**	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced
Paper 3: Statistics and Mechanics

Sample Assessment Material for first teaching September 2017

Time: 2 hours

Paper Reference **9MA0/03**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- There are **two** sections in this question paper. Answer **all** the questions in Section A and **all** the questions in Section B.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ▶

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SECTION A: STATISTICS

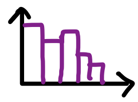
Answer ALL questions. Write your answers in the spaces provided.

1. The number of hours of sunshine each day, y, for the month of July at Heathrow are summarised in the table below.

Hours	$0 \leqslant y < 5$	$5 \leqslant y < 8$	$8 \leqslant y < 11$	$11 \leqslant y < 12$	$12 \leqslant y < 14$
Frequency	12	6	8	3	2

A histogram was drawn to represent these data. The $8 \leqslant y < 11$ group was represented by a bar of width 1.5 cm and height 8 cm.

(a) Find the width and the height of the $0 \leqslant y < 5$ group.



a) Frequency Density = $\frac{\text{Frequency}}{\text{Width}}$ and $\frac{\text{Freq Density 1}}{\text{Freq Density 2}} = \frac{\text{Height 1}}{\text{Height 2}}$

Freq. Density 1: $8 \leq y < 11$, Freq : 8, Width = $11-8=3$

Freq. Density 2 : $0 \leq y < 5$, Freq : 12, Width = $5-0 = 5$

Freq. Density 1 = $\frac{8}{3}$ and Freq. Density 2 = $\frac{12}{5}$

$\Rightarrow \frac{8/3}{12/5} = \frac{8}{H_2} \Rightarrow H_2 = \frac{12/5 \cdot 8}{8/3}$ 1 $= \underline{\underline{7.2 \text{ cm}}}$ 1

Width of $8 \leq y < 11 = 1.5\text{cm} : 3$

$\Rightarrow 1.5/3 : 1$

$0 \leq y < 5 \Rightarrow \frac{1.5}{3} \times 5 = \frac{5}{2} : 5$ $\Rightarrow$ Width is $\underline{\underline{2.5\text{cm}}}$ 1

$\frac{5}{2}$ " 2.5

b) Mean : Standard Deviation :

we need to find the midpoint of each interval :

2.5 , 6.5 , 9.5 , 11.5 , 13 (midpoints for calculator) 1

12 , 6 , 8 , 3 , 2 (frequencies) Mean = $\underline{\underline{6.63 \text{ cm}}}$ 1 and Standard deviation 1

$= 3.\underline{69}\text{cm}$

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Question 1 continued

c)

Heathrow : $\mu = 6.63$ Hurn : $\mu = 5.98$

$\sigma = 3.69$ $\sigma = 4.12$

For a lower standard deviation, data will be more consistent.

=> The number of hours of daily sunshine is more consistent at (1) Heathrow, but Hurn is further South than Heathrow, so therefore Thomas' belief is not supported. (1)

d) For Heathrow, $\mu = 6.63$ and $\sigma = 3.69$.

1 standard deviation above the mean : $\mu + \sigma = 6.63 + 3.69 = 10.32$.

Since $11 > 10.32$, all the observations in the $11 \le y < 12$ and $12 < y \le 14$ must be greater than 10.32 (3 and 2 observations respectively).

• $8 \le y < 11$, we need to estimate how many observations are in this group and are greater than 10.32.

(1)

$\frac{11 - 10.32}{3} \times 8 = 1.8$ => we estimate that there is 1.8 observations which are greater than 10.32. => $1.8 + 3 + 2 = 6.8 = 7$ days (2)

e) $N(6.6, 3.7^2)$, $\mu = 6.6$, $\sigma = 3.7$

=> $Z = \frac{X - \mu}{\sigma} = \frac{10.32 - 6.6}{3.7} = 1.0054$, $P(X > 10.32) = P(X > 1)$

$= 1 - P(X \le 1) = 1 - 0.841...$

$= 0.159$. (1)

=> Number of days $= 31 \times 0.159 = 4.9$ days (1)

f) Part d: $\mu = 6.8$ days, Part e: $\mu = 4.9$ days. $6.8 \ne 4.9$ => The model is not suitable. (1)

2. A meteorologist believes that there is a relationship between the daily mean windspeed, w kn, and the daily mean temperature, t °C. A random sample of 9 consecutive days is taken from past records from a town in the UK in July and the relevant data is given in the table below.

t	13.3	16.2	15.7	16.6	16.3	16.4	19.3	17.1	13.2
w	7	11	8	11	13	8	15	10	11

The meteorologist calculated the product moment correlation coefficient for the 9 days and obtained $r = 0.609$

(a) Explain why a linear regression model based on these data is unreliable on a day when the mean temperature is 24 °C
(1)

(b) State what is measured by the product moment correlation coefficient.
(1)

(c) Stating your hypotheses clearly test, at the 5% significance level, whether or not the product moment correlation coefficient for the population is greater than zero.
(3)

Using the same 9 days a location from the large data set gave $\bar{t} = 27.2$ and $\bar{w} = 3.5$

(d) Using your knowledge of the large data set, suggest, giving your reason, the location that gave rise to these statistics.
(1)

a) Linear Regression requires us to extrapolate data.
We know that we have to extrapolate since no data is given for 24°C. We know that extrapolating can be unreliable, hence a linear regression will be unreliable 1

b) Product Moment Correlation Coefficient measures the linear association between two Variables.
In our case, we have the linear association between daily mean windspeed, w and the daily mean temperature, t. 1

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Question 2 continued

c) $\alpha = 0.05$

One sided Test

$H_0 : \rho = 0$ v.s. $H_1 : \rho > 0$ 1

* Critical Value *

• Product moment correlation data table in the formula sheet.

$n = 9$

$\alpha = 0.05$ => From table, the critical value = 0.5822 1

At the begining, we were told that $r = 0.609$.

=> $r = 0.609 > 0.5822$ => we reject H_0, and this means that we can conclude that the product moment correlation coefficient is greater than zero. 1

d)

Large data set :

- this will not be the UK since the temperature is too high
- Options are Perth, Beijing or Jacksonville
- exclude Perth since its July, it will be winter there and thus it won't be that hot
- Wind is low (3.5) => Not near the sea.

=> We suggest that the location is Beijing. 1

(Total for Question 2 is 6 marks)

3. A machine cuts strips of metal to length L cm, where L is normally distributed with standard deviation 0.5 cm.

Strips with length either less than 49 cm or greater than 50.75 cm **cannot** be used.

Given that 2.5% of the cut lengths exceed 50.98 cm,

(a) find the probability that a randomly chosen strip of metal **can** be used.

(5)

Ten strips of metal are selected at random.

(b) Find the probability fewer than 4 of these strips **cannot** be used.

(2)

A second machine cuts strips of metal of length X cm, where X is normally distributed with standard deviation 0.6 cm

A random sample of 15 strips cut by this second machine was found to have a mean length of 50.4 cm

(c) Stating your hypotheses clearly and using a 1% level of significance, test whether or not the mean length of all the strips, cut by the second machine, is greater than 50.1 cm

(5)

a) Random Variable $L \sim N(\mu, 0.5^2)$, $\sigma = 0.5$

$P(L > 50.98) = 0.025$ (1)

$\boxed{1}$ Find the mean (μ)
$\boxed{2}$ Find the probability ($P(49 < L < 50.75)$)

Z-Score $= \dfrac{x-\mu}{\sigma}$ $\Rightarrow 1.96 = \dfrac{50.98-\mu}{0.5} \Rightarrow 50.98 - \mu = 1.96 \times 0.5$

$1 - 0.025 = 0.9750$ → look on table / or do on calculator → $= 1.96$ (1)

$\mu = 50.98 - 1.96 \times 0.5$

$\mu = 49.97 = \underline{50}$ cm (1)

(1)

$\Rightarrow L \sim N(50, 0.5^2) \Rightarrow P(49 < L < 50.75) = P(L < 50.75) - P(L < 49)$

$P(L < 50.75) = P\left(Z < \dfrac{50.75-50}{0.5}\right) = P(Z < 1.5) = \Phi(1.5) = 0.9332$

$P(L < 49) = P\left(Z < \dfrac{49-50}{0.5}\right) = P(Z < -2) = 1 - \Phi(2) = 0.0228$

$\Rightarrow P(49 < L < 50.75) = 0.9332 - 0.0228 = \underline{0.910}$ (1)

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Question 3 continued

b) The probability that a strip cannot be used will be equal to
$1 - 0.910$ * (0.910 was our answer to part a)

Now, if we let X be a random variable which denotes the number of strips that cannot be used then we're going to have that X is binomially distributed, with $n = 10$ and $p = 0.09$. $(1 - 0.910 = 0.09)$ *

$\Rightarrow X \sim B(10, 0.09)$ (1). $\Rightarrow P(X \leq 3) = 0.99$ (1)

c) $n = 15$, $\sigma = 0.6\text{cm}$, Sample mean $\bar{x} = 50.4\text{cm}$

$H_0 : \mu = 50.1\text{cm}$ v.s. $H_1 : \mu > 50.1\text{cm}$ (1) (one-sided test)

Standard error of the mean : $\sigma / \sqrt{n} = \dfrac{0.6}{\sqrt{15}}$

$\Rightarrow \bar{X} \sim N\left(50.1, \dfrac{0.6^2}{15}\right)$ (1)

$\Rightarrow P(\bar{X} > 50.4) = P\left(Z > \dfrac{50.4 - 50.1}{0.6/\sqrt{15}}\right) = P(Z > 1.94) = 1 - \Phi(1.94)$
$= 0.026$ (p-value) (1)

$\Rightarrow 0.026 > 0.01 = \alpha$ (1)

$\Rightarrow$ Do not reject H_0 and we can conclude that there is insufficient evidence that the mean length of the strips is greater than 50.1cm. (1)

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$ **(2)**

(b) Explain why the events A and B are not independent. **(1)**

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region. **(5)**

(d) Find $P([B \cup C]')$ **(2)**

a) Find $P(A'|B')$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

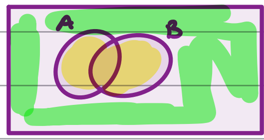
$\Rightarrow P(A'|B') = \dfrac{P(A' \cap B')}{P(B')}$

$= \dfrac{0.33}{0.55} = 0.6$ (1)

$P(B') = 1 - 0.45 = 0.55$

$P(B) = 0.45$

$P(A' \cap B') \Rightarrow$



$\Rightarrow P(A'|B') = 0.6$ (1)

$P(A \cup B) = 0.35 + 0.45 - 0.13 = 0.67$

$\Rightarrow P(A' \cap B') = 1 - 0.67 = 0.33$

b) If the event A and the event B are independent, then $P(A) \cdot P(B) = P(A \cap B)$. So, for our case we have the required information, so we can compute it to see if they are independent.

$\Rightarrow P(A \cap B) = 0.13$, $P(A) = 0.35$, $P(B) = 0.45$ $\Rightarrow P(A) \cdot P(B) = 0.35 \times 0.45$
$= 0.1575 \neq 0.13$

$\Rightarrow$ Since $P(A \cap B) \neq P(A) \cdot P(B)$, A and B are not independent. (1)

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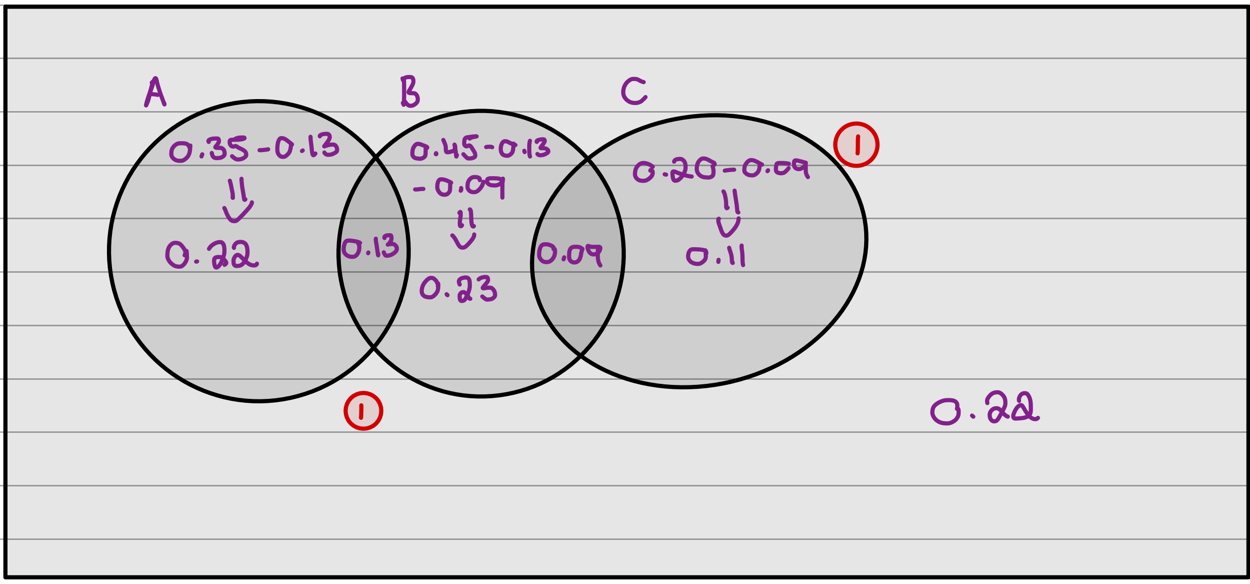
Question 4 continued

c) • Mutually exclusive => two events cannot happen simultaneously => A and C cannot happen simultaneously.

• Statistically independent = independent => B and C are independent.



$P(A) = 0.35$, $P(B) = 0.45$, $P(A \cap B) = 0.13$, $P(C) = 0.20$

B and C are independent, hence $P(B \cap C) = P(B)P(C) = 0.45 \times 0.20 = 0.09$

$P(A \cup B \cup C) = 1 - (0.22 + 0.13 + 0.23 + 0.09 + 0.11) = 1 - 0.78 = 0.22$

d) $P([B \cup C]')$ will be everthing that is not part of B and/or C.

$\Rightarrow P([B \cup C]') = 0.22 + 0.22 = \underline{\underline{0.44}}$

(Total for Question 4 is 10 marks)

5. A company sells seeds and claims that 55% of its pea seeds germinate.

(a) Write down a reason why the company should not justify their claim by testing all the pea seeds they produce.

(1)

A random selection of the pea seeds is planted in 10 trays with 24 seeds in each tray.

(b) Assuming that the company's claim is correct, calculate the probability that in at least half of the trays 15 or more of the seeds germinate.

(3)

(c) Write down two conditions under which the normal distribution may be used as an approximation to the binomial distribution.

(1)

A random sample of 240 pea seeds was planted and 150 of these seeds germinated.

(d) Assuming that the company's claim is correct, use a normal approximation to find the probability that at least 150 pea seeds germinate.

(3)

(e) Using your answer to part (d), comment on whether or not the proportion of the company's pea seeds that germinate is different from the company's claim of 55%

(1)

a) If they test all their peas then they will have none to sell. This is not an effective business method, as they can't get any income if they have destroyed all their peas. 1

b) let S is the random variable which is the number of seeds out of 24 that germinate. $\Rightarrow S \sim B(24, 0.55)$. $n = 24$, $p = 0.55$

let T be the random variable which is the number of trays with at least 15 or more seeds germinating. $= T \sim B(10, q)$ 1

$\Rightarrow q = P(S \geq 15)$ $(n = 24, p = 0.55)$

$q = 0.299$ 1

$\Rightarrow T \sim B(10, 0.299)$

$\Rightarrow P(T \geq 5) = 0.149$ 1

Question 5 continued

c) • n is large
• P (probability) must be close to $\frac{1}{2}$ or 0.5 (1)

d) $X \sim N(\mu, \sigma^2)$

When this approximation : $\mu = n.p$ and $\sigma^2 = npq$ $n = 240, p = 0.55$
$\Rightarrow \mu = 240 \times 0.55 = 132$ and $\sigma^2 = 240 \times 0.55 \times 0.45 = 59.4$ $q = 0.45$

$\Rightarrow X \sim N(132, 59.4)$ (1) * We must use the continuity correction, so when we work out our probability, we subtract 0.5. *

$\Rightarrow P(X \geq 149.5) = P\left(Z \geq \frac{149.5 - 132}{\sqrt{59.4}}\right)$ (1) $= P(Z \geq 2.27) = \underline{\underline{0.0116}}$ (1)

e) Probability from part d was 0.0116, which is a very small number. This small number tells us that there is evidence to suggest that the company's claim is incorrect. (1)

TOTAL FOR SECTION A IS 50 MARKS

SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

6. At time t seconds, where $t \geqslant 0$, a particle P moves so that its acceleration $\mathbf{a}\text{ m s}^{-2}$ is given by

$$\mathbf{a} = 5t\mathbf{i} - 15t^{\frac{1}{2}}\mathbf{j}$$

When $t = 0$, the velocity of P is $20\mathbf{i}\text{ m s}^{-1}$

Find the speed of P when $t = 4$

(6)

$\underline{a} = 5t\underline{i} - 15t^{1/2}\underline{j}$, $t = 0$, $\underline{v} = 20\underline{i}\text{ ms}^{-1}$

$\downarrow v?$ Speed when $t = 4$?

Displacement, Velocity, Acceleration — Differentiate $\downarrow$, Integrate $\uparrow$

We can integrate our equation for acceleration with respect to t, to find an expression for the velocity.

$\Rightarrow \underline{a} = 5t\underline{i} - 15t^{1/2}\underline{j}$ $\quad \int 5t\,dt = \frac{5t^2}{2}$ and $\int 15t^{1/2}\,dt = 10t^{3/2}$

$\Rightarrow \int 5t\underline{i} - 15t^{1/2}\underline{j}\,dt = \frac{5t^2}{2}\underline{i} - 10t^{3/2}\underline{j} + C = \underline{V}$ (1) (1)

When $t = 0$, $\underline{V} = 20\underline{i} \Rightarrow \frac{5(0)^2}{2}\underline{i} - 10(0)^{3/2}\underline{j} + C = 20\underline{i} \Rightarrow C = 20\underline{i}$

$\Rightarrow \underline{v} = \frac{5t^2}{2}\underline{i} - 10t^{3/2}\underline{j} + 20\underline{i}$ (1)

When $t = 4$, we need to find velocity $\Rightarrow \underline{V} = \frac{5(4)^2}{2}\underline{i} - 10(4)^{3/2}\underline{j} + 20\underline{i} = 40\underline{i} - 80\underline{j} + 20\underline{i}$

$\Rightarrow \underline{V} = 60\underline{i} - 80\underline{j}$ (1) (Velocity expression for $t=4$)

We can now find the speed by working out the magnitude since speed is the scaler equivalant of velocity.

Speed $= |\underline{V}| = \sqrt{(60)^2 + (-80)^2} = \underline{100\text{ ms}^{-1}}$ (1) (1)

Question 6 continued

(Total for Question 6 is 6 marks)

7. A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$.

A particle of mass m is placed on the plane and then projected up a line of greatest slope of the plane.

The coefficient of friction between the particle and the plane is μ.

The particle moves up the plane with a constant deceleration of $\frac{4}{5}g$.

(a) Find the value of μ.

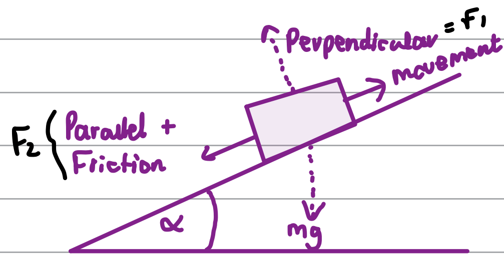
(6)

The particle comes to rest at the point A on the plane.

(b) Determine whether the particle will remain at A, carefully justifying your answer.

(2)

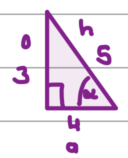
a) μ : Coefficient of Friction



$\tan\alpha = \frac{3}{4}$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = 3/5$



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$F_1 = mg\cos\alpha$ (1)

$F_1 = \frac{4}{5}mg$

$F_2 = \text{Friction} + \text{Parallel Force}$ (1)

$F_2 = \mu F_1 + mg\sin\alpha$ (1)

$F_2 = \mu\left(\frac{4}{5}mg\right) + \frac{3}{5}mg$

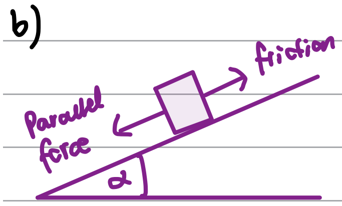
$\Rightarrow \frac{4}{5}mg = \mu\left(\frac{4}{5}mg\right) + \frac{3}{5}mg$ (1) (mg cancels out)

$\Rightarrow \frac{4}{5} = \frac{4}{5}\mu + \frac{3}{5}$

(1)

$\Rightarrow \frac{4}{5}\mu = \frac{4}{5} - \frac{3}{5} = \frac{1}{5} \Rightarrow \mu = \frac{1}{5} \times \frac{5}{4} \Rightarrow \underline{\underline{\mu = \frac{1}{4}}}$ (1)

b)



At rest friction will be in the upward direction

$\text{Friction} = \mu mg\cos\alpha = \frac{1}{4}\cdot\frac{4}{5}mg = \underline{\underline{\frac{1}{5}mg}}$

$\text{Parallel Force} = mg\sin\alpha = \underline{\underline{\frac{3}{5}mg}}$

(1)

$\frac{3}{5}mg > \frac{1}{5}mg$, Parallel force is greater than the frictional force, so then the particle will start to move down the slope and it will no longer remain at A.

(1)

Question 7 continued

(Total for Question 7 is 8 marks)

8. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively*]

A radio controlled model boat is placed on the surface of a large pond.

The boat is modelled as a particle.

At time $t = 0$, the boat is at the fixed point O and is moving due north with speed 0.6 m s^{-1}.

Relative to O, the position vector of the boat at time t seconds is **r** metres.

At time $t = 15$, the velocity of the boat is $(10.5\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.

The acceleration of the boat is constant.

(a) Show that the acceleration of the boat is $(0.7\mathbf{i} - 0.1\mathbf{j})$ m s^{-2}.

(2)

(b) Find **r** in terms of t.

(2)

(c) Find the value of t when the boat is north-east of O.

(3)

(d) Find the value of t when the boat is moving in a north-east direction.

(3)

a) We want to find the acceleration from $t = 0s$ and $t = 15s$.

$\underline{r} = X$

$\underline{u} = 0.6\underline{j}$ ms^{-1}

$\underline{v} = (10.5\underline{i} - 0.9\underline{j})$ ms^{-1}

$\underline{a} = ?$

$t = 15s$ (Scaler)

$v = u + at$

$10.5\underline{i} - 0.9\underline{j} = 0.6\underline{j} + 15\underline{a}$ 1

$10.5\underline{i} - 0.9\underline{j} - 0.6\underline{j} = 15\underline{a}$

$10.5\underline{i} - 1.5\underline{j} = 15\underline{a}$

$\underline{a} = (0.7\underline{i} - 0.1\underline{j})$ ms^{-2} 1

b) We want to find $\underline{r}$ in terms of t.

$\underline{r} = ?$

$\underline{u} = 0.6\underline{j}$

$\underline{v} = 10.5\underline{i} - 0.9\underline{j}$

$\underline{a} = 0.7\underline{i} - 0.1\underline{j}$

$t = ?$

$\underline{r} = \underline{u}t + \frac{1}{2}\underline{a}t^2$ 1

$\underline{r} = 0.6\underline{j}t + \frac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$ 1

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Question 8 continued

c) North East => the vectors $\underline{i}$ and $\underline{j}$ will be equal in the expression for displacement. 1

$$\Rightarrow \quad \underline{r} = 0.6\underline{j}t + \tfrac{1}{2}(0.7\underline{i} - 0.1\underline{j})t^2$$
$$= 0.6\underline{j}t + (0.35\underline{i} - 0.05\underline{j})t^2$$
$$= 0.6\underline{j}t + 0.35\underline{i}t^2 - 0.05\underline{j}t^2$$
$$\underline{r} = t(\underbrace{0.6\underline{j}} + \underbrace{0.35\underline{i}t} - \underbrace{0.05\underline{j}t})$$

$\underline{j}$ component $\Rightarrow$ $0.6 - 0.05t = 0.35t$ ← $\underline{i}$ component 1

$\Rightarrow 0.6 = 0.4t$

$\Rightarrow$ $\underline{\underline{t = 1.5\text{s}}}$ 1

d) $\underline{v} = \underline{u} + \underline{a}t$

$\underline{v} = 0.6\underline{j} + (0.7\underline{i} - 0.1\underline{j})t$ 1

Now, we want to set the $\underline{i}$ and $\underline{j}$ components of $\underline{v}$ equal to each other.

$$\Rightarrow \underline{v} = \underbrace{0.6\underline{j}} + \underbrace{0.7\underline{i}t} - \underbrace{0.1\underline{j}t}$$

$\Rightarrow$ $0.6 - 0.1t = 0.7t$ 1 $\Rightarrow$ $0.6 = 0.8t$

$\underline{\underline{t = 0.75\text{s}}}$ 1

(Total for Question 8 is 10 marks)

9.

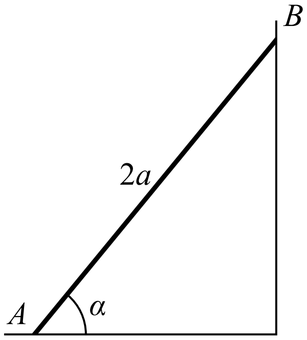


Figure 1

A uniform ladder AB, of length $2a$ and weight W, has its end A on rough horizontal ground.

The coefficient of friction between the ladder and the ground is $\frac{1}{4}$.

The end B of the ladder is resting against a smooth vertical wall, as shown in Figure 1.

A builder of weight $7W$ stands at the top of the ladder.
To stop the ladder from slipping, the builder's assistant applies a horizontal force of magnitude P to the ladder at A, towards the wall.
The force acts in a direction which is perpendicular to the wall.

The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan \alpha = \frac{5}{2}$.

The builder is modelled as a particle and the ladder is modelled as a uniform rod.

(a) Show that the reaction of the wall on the ladder at B has magnitude $3W$.

(5)

(b) Find, in terms of W, the range of possible values of P for which the ladder remains in equilibrium.

(5)

Often in practice, the builder's assistant will simply stand on the bottom of the ladder.

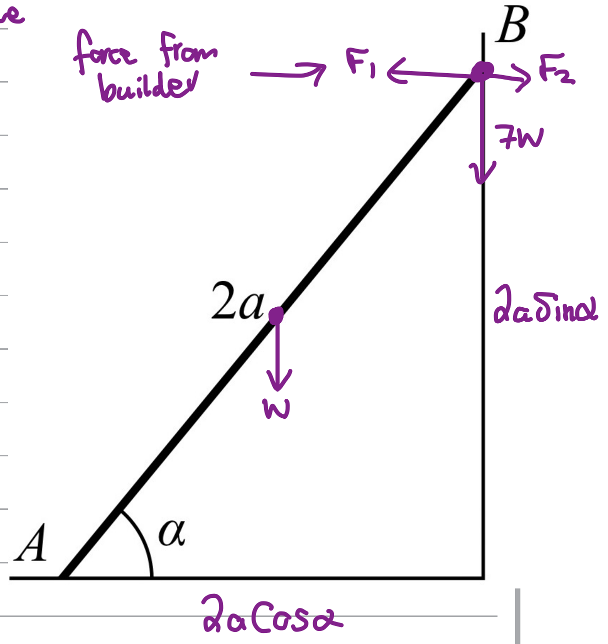
(c) Explain briefly how this helps to stop the ladder from slipping.

(3)

DO NOT WRITE IN THIS AREA

Question 9 continued

Moments: at point A there is a pivot point and we have multiple forces acting on A, and they are all in equilibrium since there is no movement.



=> Take moments about A, (1) and since all the forces are in equilibrium, their sum will add to 0.

• the builder • weight of ladder • Push back force

=> $7W \cdot 2a\cos\alpha + Wa\cos\alpha - S \cdot 2a\sin\alpha = 0$ (2)

=> $14Wa\cos\alpha + Wa\cos\alpha - 2Sa\sin\alpha = 0$

=> $15Wa\cos\alpha = 2a \cdot S \cdot \sin\alpha$

=> $15W \cdot \cos\alpha = 2S \cdot \sin\alpha$

=> $15W = 2 \cdot S \cdot \dfrac{\sin\alpha}{\cos\alpha}$ $\left(\tan\alpha = 5/2 = \dfrac{\sin\alpha}{\cos\alpha}\right)$

=> $15W = 2 \cdot S \cdot 5/2$ (1)

=> $15W = 5 \cdot S$ => $\underline{S = 3W}$ as required. (1)

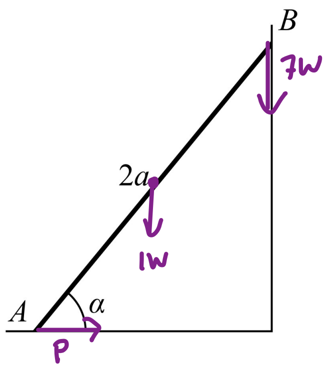
b) Perpendicular Force $= 1W + 7W = 8W$ (F_1) (1)

Friction $= \mu F_1 = 1/4 \times 8 = 2W$ (1)

3W comes from part a of the q.

=> Max $P = 3W + 2W = 5W$ (1)

=> Min $P = 3W - 2W = 1W = W$ (1)



=> The range of P will be : $\underline{W \le P \le 5W}$ (1)

c) By standing on the bottom of the ladder this helps to stop the ladder from slipping since :

• by taking moments at A, the reaction on the ladder at B is unchanged. (1)

• More weight on the ladder, so the perpendicular force (F_1) will increase (1)

• Since F_1 increases, friction will increase since friction $= F_1 \cdot \mu$. (1)

Question 9 continued

Question 9 continued

(Total for Question 9 is 13 marks)

10.

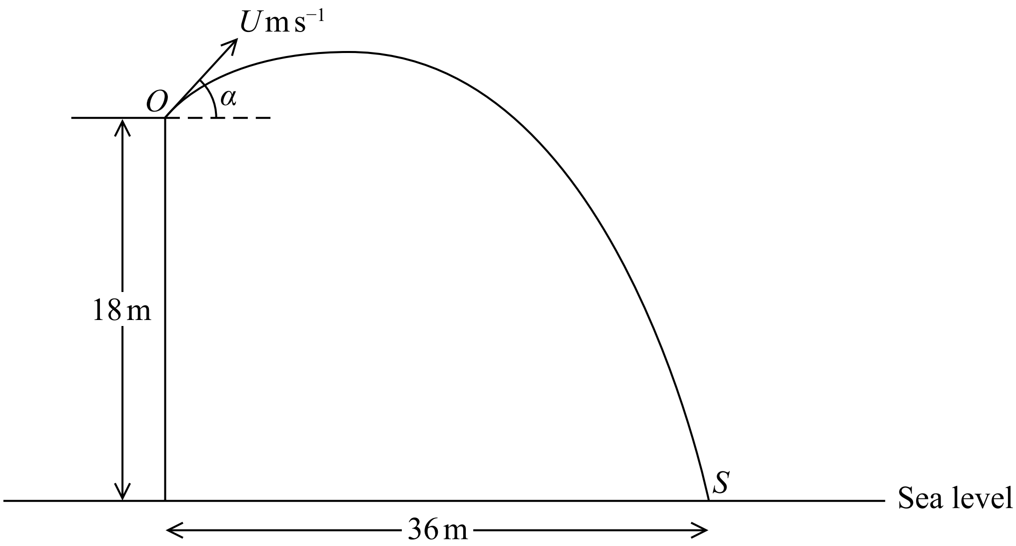


Figure 2

A boy throws a stone with speed U m s^{-1} from a point O at the top of a vertical cliff.
The point O is 18 m above sea level.

The stone is thrown at an angle α above the horizontal, where $\tan \alpha = \frac{3}{4}$.

The stone hits the sea at the point S which is at a horizontal distance of 36 m from the foot of the cliff, as shown in Figure 2.
The stone is modelled as a particle moving freely under gravity with $g = 10$ m s^{-2}

Find

(a) the value of U,

(6)

(b) the speed of the stone when it is 10.8 m above sea level, giving your answer to 2 significant figures.

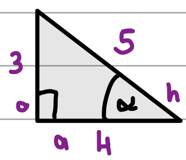
(5)

(c) Suggest two improvements that could be made to the model.

(2)

a) U is the initial Velocity that the Stone was thrown at.

$\tan \alpha = 3/4$



SOHCAHTOA $\Rightarrow$ $\cos\alpha = 4/5$, $\sin\alpha = 3/5$

We can split the motion of the stone into its vertical component and its horizontal component.

DO NOT WRITE IN THIS AREA

Question 10 continued

Horizontal : $S = 36m$ $\Rightarrow$ $S = ut$ (1)

$u = U \cdot \cos\alpha$ $\quad 36 = U\cos\alpha \cdot t$ (1)

$V =$ $\quad 36 = 4/5 \cdot Ut$

$a = 0\ ms^{-2}$ $\quad 45 = Ut \Rightarrow t = \frac{45}{u}$

$t = ?$

Vertical : $S = -18m$ $\quad S = ut + \frac{1}{2}at^2$ (1)

$u = U\sin\alpha = U \cdot 3/5$ $\quad -18 = \frac{3}{5}U \cdot \frac{45}{U} + \frac{1}{2}(-10)\left(\frac{45}{u}\right)^2$ (1) (1)

$V = X$

$a = -10\ ms^{-2}$ $\quad -18 = 27 - \frac{10125}{u^2}$

$t = ?\ \frac{45}{u}$

$\Rightarrow 10125 = 45u^2$

$\Rightarrow U = \sqrt{\frac{10125}{45}} = 15\ ms^{-1}$ (1)

$\Rightarrow U = 15ms^{-1}$

b) Vertical : $S = -18 + 10.8 = -7.2m$ $\quad V^2 = U^2 + 2as$

$U = U\sin\alpha = 15 \cdot 3/5 = 9ms^{-1}$ $\quad V^2 = (9)^2 + 2(-10)(-7.2)$ (1)

$V = ?$ $\quad V = \sqrt{225}$

$a = -10\ ms^{-2}$ $\quad \Rightarrow V = 15ms^{-1}$ (1)

Horizontal : $V = U\cos\alpha = 15 \times 4/5 = 12ms^{-1}$ (1)

Go from two vector components of velocity to a scalar which is speed by finding magnitude of the velocity vector.

$\Rightarrow$ Speed $= \sqrt{15^2 + 12^2} = 19.209\ldots$ (1) $\Rightarrow$ Speed when the stone is 10.8m above sea level will be $19ms^{-1}$ (1)

c)

- take into account air Resistance (1)
- what effect the wind has on the motion of the stone. (1)

Question 10 continued

Question 10 continued

(Total for Question 10 is 13 marks)

TOTAL FOR SECTION B IS 50 MARKS
TOTAL FOR PAPER IS 100 MARKS